FERMAT Salutes: V.G. Veselago

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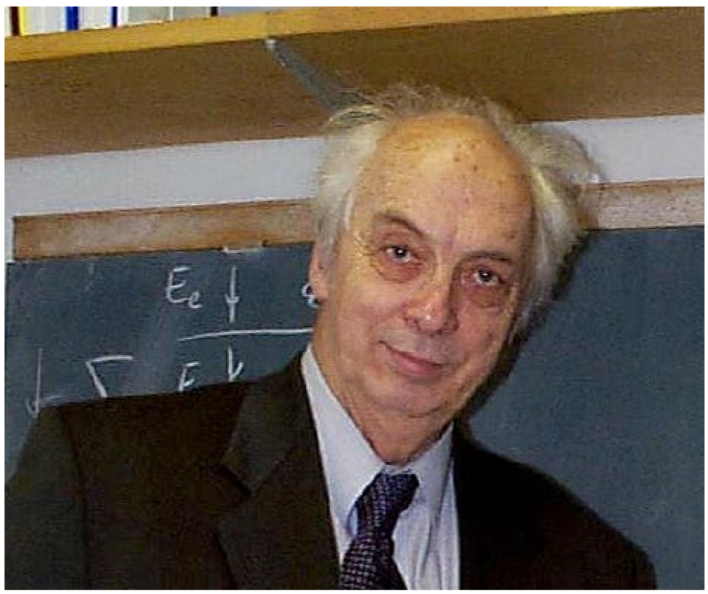
This article is dedicated to Prof. Victor G. Veselago whose imaginative ideas have predicated the advent of metamaterial concepts and inspired a generation of researchers working in diverse fields of physics, optics and engineering.



The words *metamaterials* and *negative refraction* are nowadays inseparable from the name of Victor Veselago. They are familiar to not only physicists and electrical engineers but also frequently appear in the news and mass media. Immense advances in the field of artificial electromagnetic materials have turned the magic effects, perceived in the past nothing more than science fiction, into reality. The flat lenses with "super resolution" and "invisibility cloaks", promising ubiquitous practical applications far beyond their native fields of optics and electromagnetics, have caught enormous public interest. Although an exuberant publicity causes certain controversy in the scientific community, the researchers are very enthusiastic in their relentless efforts to uncover and exploit novel phenomena brought about by the modern artificial functional materials.

The current activities in the field of metamaterials and their applications have been predicated by Victor Veselago's work published nearly half a century ago. In his seminal paper "The electrodynamics of substances with simultaneously negative values of ε and μ" published in *Sov. Phys. Usp.* **10**, 4, 509 (1968), he had consistently introduced and justified the concept of the negative refractive index, n. He had not proposed a magic material with $n < 0$ but systematically described the wave properties in such a medium if it existed. He has revealed the basic relations between the negative n and negative permittivity and permeability, the phase and group velocities, Snell's law, Doppler and Cherenkov effects. This comprehensive study proved to be extremely influential and, according to

Google, it has attracted more than 8220 citations to date. Veselago's fundamental work has laid a solid conceptual foundation for the negative refraction paradigm and underpinned the later works by Sir John Pendry[1] and Prof. David Smith[2] with colleagues who have incepted the physical realisations of the artificial media with the negative refractive index and produced the crucial experimental demonstrations of this kind of refraction.

Prof. V. Veselago belongs to a generation of Soviet physicists who were educated shortly after the Second World War. It was a difficult time for the country healing its war wounds. Nevertheless, the high spirit and enthusiasm of the society at the time were exceedingly encouraging for ambitious students to pursue a research career. It was a strong belief that the country recovery and future prosperity would be geared up by the next generation of bright scientists trained and nourished at the heart of the Soviet science. Therefore, from the outset many students had placements and part-time jobs in the Academy of Science laboratories and worked under guidance of the leading Soviet researchers. Victor Veselago followed this path and after graduating from Moscow State University in 1952, he started his work at the P.N. Lebedev Physical Institute of the Academy of Sciences (FIAN) in Moscow. Here he had performed his fundamental studies of the negative refraction. Since 1983 he has been a Head of the Magnetic Materials Laboratory in A.M. Prokhorov Institute of General Physics of the Academy of Science.

I met V. Veselago first time in 1985 at the School on Spin and Magnetostatic Waves held near the Turkmen capital Ashgabat. It was the time of great excitement about spin waves in ferrite films with negative effective permeability and magnetoplasmons in solid-state plasmas with negative permittivity. V. Veselago was tirelessly searching for materials, which would combine the two worlds and exhibit the negative permittivity and permeability simultaneously. Therefore, his focus was on magnetic semiconductors that seemed to be potential candidates for obtaining the negative refractive index. Alas, the available materials did not perform as he wished, yet….

Next time we met nearly 20 years later. By this time, Veselago's imaginative ideas have come to fruition after the ground-breaking works of J. Pendry[1] and D.Smith[2]. And now they still fuel this rapidly growing field of metamaterials and their applications. Prof. Veselago's work has been widely recognised as a cornerstone in our understanding of the electromagnetic and optical phenomena[3]. For his contribution to these fields he has been awarded the 2009 C E K Mees Medal by the Optical Society of America (OSA) and the 2004 V.A. Fock Prize (Russia).

Prof. V. Veselago is actively involved with the metamaterial and optical communities in many capacities and often presents his talks at the major international conferences. His achievements and his whole life are an inspirational example for all of us, those who had a privilege to know him for many years, and especially for young people who know his name mainly from papers.

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My road to negative refraction

V.G. Veselago

Childhood – Dnepr, Volga, Moscow

I was born on the 13th June 1929. According to my birth certificate, my birthplace was called the village Kichkas in Zaporozhye region. It was in the Soviet Republic of Ukraine at that time, and now it would be an independent state of Ukraine. You would not find this village on the modern map of Ukraine, though. It disappeared when the Dnepr Hydropower Station (DniproHES) was built. The village submerged under the water reservoir created by the dam. Perhaps, the only opportunity for me to visit my birthplace nowadays would be to sail over the sunken village in a boat or a cruise ship. Unfortunately as an adult, I have never visited Zaporozhye, the nearest city to the DnerpoHES dam.

My father worked as an engineer at the construction of the power station, and after its completion in 1932, my family moved out and I barely have any memories of either Dnepr or the Power Station. For his contribution to building DniproHES, my father was decorated with the Order of Lenin – one of the highest State honours in the USSR, and was granted a three-room apartment in Moscow. In addition, he was appointed a General Manager of a new Sredne-Volzhsky Hydropower Station to be constructed at the river Volga. My father was very proud of the fact that he was trusted to lead this huge project and looked forward to building the entire plant from the beginning to the end.

This new Hydroelectric Station was planned near the town of Yaroslavl, just north of the famous Tolga monastery. The monastery was already closed at the time, and its churches and premises were occupied by various construction offices and used also for accommodation for the engineers working at the building site. Our family, my father, Georgiy Sergeyevich, mother, Elena Borisovna, older brother, Andrey, and myself, were allocated two rooms in the so-called “red house”, the former monastery hostel. The same building housed the family of my uncle, the engineer Vyacheslav Alekseevich Zacharyevsky, who was married to my mother’s sister, Maria Borisovna. He and my father were friends, and all of us were essentially a one big family.

My first recollections of my childhood come from the time of our life in Tolga. The mighty river Volga, beautiful cruise ships of that time, the ice melting and crushing loudly in the spring, they all left unforgettable memories in my mind. Our quiet life did not last long, however, and was suddenly interrupted in August 1935, when my father came from work and announced that the construction was halted and that we were moving to Moscow. I vigorously protested to this news - I sat down on the floor and started crying. But this did not help and within about a week, our family packed in my father's work car and we headed to Moscow. My father and his chauffer were taking turns driving the car, while my mother, brother, and I sat at the back. We left Tolga early in the morning and arrived to Moscow late at night, in the dark. Finally, we moved in the apartment granted to my father after the completion of DniproHES.

Later, I found out the reasons behind termination of the already well progressed Yaroslavl HES. The disruption was caused by interference of the almighty NKVD who supplied a free labour to various construction sites. The NKVD was overseeing many thousands of convicts, who by and large were innocent people ruthlessly exploited by the authorities. At the time, there was already the experience of using inmates at the construction of Belomor-Balt canal, and NKVD was going to bring them again to build a dam on Volga, nearby Rybinsk, just north of the planned Yaroslavl HES. This dam was to create a huge artificial "Rybinsk sea", which would have buried several towns, villages, and vast areas of arable land.

My father's career did not suffer from his relocation to Moscow, where he became a Deputy Director and Chief Engineer of the Design Institute "Glavgidroenergoproekt" ("Главгидроэнергопроект"). This organisation was responsible for the design and construction of the hydroelectric stations across the entire USSR. The successor of this Institute still exists and its offices are located in a high-rise building at the junction of Leningrad and Volokolamsk highways in Moscow.

In Moscow, my father fully devoted himself to his new work, albeit from time to time he regretted that he did not have opportunity to complete his Volga project. Our happy life in Moscow was suddenly shattered in September 1937, when my father was killed in a tragic railway accident. On his way back home from the Russian Far East, where he carried out the assessment of the hydroelectric resources, his courier train collided with a freight train. My father was the only passenger casualty who died along with only one member of the train crew in this catastrope.

A year after my father's death, I started school. I was admitted straight to the second grade because I was able to read and write fluently at the time. I was very lucky to study in an excellent school with superb teachers and a wonderful headmaster, Fyodr Fyodorovich Roschin, to whom I am greatly indebted for his help.

At school, I had rather average marks and tried to avoid teacher's attention and comments should they be positive or negative.

The Second World War. "What is a radio?"

Almost four years passed, and then our life changed drastically again as the war came to the USSR in 1941. When the German armies advanced to Moscow, the city was mass evacuated, moving people and factories to the East. My mother and me together with my aunt and her husband, Vyacheslav Alekseevich Zacharyevsky were relocated to Tashkent (Uzbekistan). We returned to Moscow only at the spring of 1944. At that time, I had missed 3 months of school, and only 3 weeks remained to the annual exams. Several schools in our area refused to enrol me, saying that I should repeat the year. The situation was further aggravated by the fact that the school which I had attended before the war was closed after the war. However, my mother managed to find the headmaster of my former school, F. F. Roschin. It turned out that he was now the headmaster of an elite school for the children of the Moscow officialdom located in the centre of Moscow. F. F. Roschin remembered me, as well as my then late brother Andrey, who was at the same school. Andrei finished the school with distinction before the war, became a lieutenant of the Red Army, and died from his wounds at a war hospital in 1943. Roschin's decision about me was instant – come to the school and prepare for the exams now…. Then we would see what to do next.

I successfully passed all of my exams with the grades of 4 and 5 (out of 5, "good" and "excellent") and was admitted to the next year studies. Then an incidental mishap seen insignificant at the time has essentially predicated my future. Namely, at the beginning of the new academic year, I fell ill (I can't remember what it was) and, following the then customary treatment, I was prescribed to be in bed for a while. The bed regime was exceptionally boring, so I continued my favourite activities – digging through an extensive home library left behind by my late father. Among various technical and fiction books, an average size book entitled "What is a radio?" caught my eye. The author of the book was someone called S. Kin. The book turned out to be fairly comprehensible and described in detail how to make a simple radio receiver. I instantly wondered whether I could build such a radio myself.

The same evening, I asked my mother to buy several simple parts at the local shop, and I assembled the radio following the instructions detailed by S. Kin. The next day I was already listening to some concert broadcasted by the Moscow radio station. This was the beginning of my ham radio hobby, which lasted for nearly 3 years, till the end of my schooling.

Just before finishing the school, I started thinking about university. I had a firm intention to pursue my studies at an institution with a radio engineering orientation, and even looked at the available information about the respective Moscow institutes. I must say that I was contemplating about a career of radio engineer at some plant and never even could imagine myself of becoming a scientist. Moreover, I probably would not be able even to define what a “researcher” meant. However, another incidental turn essentially predetermined my destiny.

One of my classmates showed me an information booklet with the announcement of a new faculty specialising in physics and technology at Moscow State University. This new faculty was supposed to educate specialists for various research institutes with different specialisations, including Radiophysics. This new faculty had also offered an attractive opportunity of sitting the entrance exams a month earlier than in the other universities around the country where they began on the 1st August. This would give a chance of the second attempt at another institution in the case of failure here. This contingency option was the main reason why I applied to this new faculty. Altogether, I had to sit the following 9 exams:

- Algebra (written and oral),
- Geometry and trigonometry (written and oral),
- Physics (written and oral),
- Chemistry,
- German Language, and
- Essay in Russian Language and Literature.

Those who passed these exams successfully would be invited for the interview with the specially-appointed panel at the final stage.

On the 1st July, I sat the written exam in Algebra and miserably failed it - I got a grade 2 (out of 5). The reason for this fiasco was simple, I was anxious in the novel environment, i.e. I was unprepared psychologically as it would be said today. The examination was at the university’s Department of Physics, in the so called “Grand Physics Auditorium”. We were seated one by one in a huge room, with the amphitheatre ascending into havens. I felt myself a tiny grain of sand in an endless space. So, I was lost and could not collect my thoughts and concentrate on solving the tasks. As the result, I failed the exam despite being fairly well prepared for it.

The day after the exam, I came to the admission committee to withdraw my documents and subsequently resubmit my application to another institute in accordance with my contingency plan. Instead, I suddenly was offered another chance here. I was told: “Listen, you haven’t got much to do for now. So, come to the oral exam the day after tomorrow, and then we will see what to do with you.”

Of course, I came for the oral Algebra exam and got the top grade 5 (out of 5) for it. Immediately, I was told to sit the remaining exams. I then got 6 grades 5 (out of 5) and only one grade 4 (out of 5) in German Language. The cause for my success was also psychological. Believing that I had nothing to lose, I had no excessive stress anymore and could concentrate on the tasks. I performed especially well at the Chemistry exam. I don’t remember what exactly I said there, but the examiner told me that I was an exceedingly brave person, and they needed people like me. Then he also told me that he would specifically refer me to the examination board.

After the exams, there was an interview. I was presented to the panel of some highly distinguished people, and one of them asked me,

- Tell us, why do you want to specialise in Radiophysics? (The faculty had other 5 specialisations.)
    - Because I have 3 year experience in ham radio and I am particularly interested in this subject.
- Tell me, which radio receiver in particular have you built?
    - Broadband four tube super heterodyne receiver
- And what type of tube did you use in the acoustic amplifier?
    - I instantly named the tube type.
- And what was the reason for your choice?
    - I recited by memory all the specifications of the tube and explained why it was the most appropriate for the purpose.
- Thank you. You may go now.

I had got an impression that both my examiner enjoyed this discussion as much as I did. Next day, I found my name in the list of students admitted to the course.

PhysTech. Crimea. FIAN

Physics and Technology Faculty of the Moscow State University (FTF MSU) was the predecessor of the Moscow Institute of Physics and Technology (MIPT). The PhysTech faculty adopted exceptionally intensive teaching approach, which not everybody could survive. The lectures were delivered by the famous scientists like

P.L. Kapitsa and L.D. Landau. The majority of lecturers held the tenured positions at the Academy of Science institutions, where the students had placements to gain the first-hand experience of the most recent research activities of the time. Interestingly, I got the top grade 5 (out of 5) for the experimental physics exam by P. L. Kapitsa, and failed the theoretical physics exam by L. D. Landau. Then I took a resit exam with E. M. Lifshitz, but failed again getting the same grade 2 (out of 5). I passed it eventually on the second resit with the grade 4 (out of 5) from V. B. Berestetzky.

After my second year of studies, I and several my classmates were offered summer placements at the Crimea Radio Astronomy Laboratory of the P. N. Lebedev Institute of Physics of the Academy of Sciences (FIAN). Obviously, I gladly accepted the offer and spent the entire summer in Crimea. I worked essentially as a radio technician, using the skills and knowledge acquired during my ham radio days. The summer placements in Crimea were repeated again after both 3rd and 4th year of my studies. The head of the Crimea FIAN laboratory at the time was the famous physicist, Semen Emmanuilovich Khaikin. The interaction with this exceptional scientist and generally incredible person was invaluable for me. Interestingly, it turned out that S. E. Khaikin has pseudonym was “S. Kin” and that he was the author of the book “What is a radio?”, which first introduced me to the radio engineering.

At the end of the summer spent with the Crimea FIAN Laboratory, I decided to continue working at FIAN, and over the winter of 1949-1950. I called Prof. Sergey Michailovich Rytov, asking for opportunity to work at the FIAN during the university winter holidays. Prof. Rytov was a formal mentor of our class at PhysTech, and he immediately invited me to meet with him at FIAN in a couple of days. A few of my classmates joined me and all of us crowded in Rytov’s office.

Rytov was a famous theoretician, so his first question to us was “Who wants to do theory?” None of us was interested to do theory. Rytov looked disappointed and said that he would need to call Sasha in this case. He left the office and came back with Alexander Mikhailovich Prokhorov, whom we all knew very well as he was teaching the physics lab, which we nicknamed “soldering practicum”. Rytov explained the situation to Prokhorov, and without any hesitation, Prokhorov pointed his finger at me and said, “This one, I’m taking him to my lab”. The others started shouting immediately what would happen to them, and all were reassured by Rytov that everyone would be duly allocated a place. Indeed, other staff members were summoned and everyone was assigned a supervisor. Thus, I became a student intern at the FIAN oscillation laboratory. The fact that Prokhorov selected me was easily understandable - he noticed me during our lab sessions in which I definitely stood out due to my radio engineering background, or rather my ham radio hobby.

At the same time, S. E. Khaikin offered me to work with him not only during the summer holidays at Crimea, but also at his Moscow radio astronomy lab during the winter. After some consideration, I declined his offer. Radio astronomy as a science seemed to me too far detached from reality and hence boring. However, after some discussions with both Khaikin and Prokhorov, it decided that I would work with Khaikin as a radio technician in Crimea during the summer and as a student intern with Prokhorov in Moscow during the winter.

At that time, Prokhorov's group started works on radio spectroscopy of gases, and my work was supervised not only by A. M. Prokhorov, but also by his close associates Mark Efremovich Zhabotinsky and a PhD student Nataliya Aleksandrovna Irisova. I was actively involved in the development of the first microwave radio spectroscopes, which were used for the studies of several gases, primarily ammonia. Ammonia was later utilised in the first lasers and masers for which A.M. Prokhorov, N.G. Basov and C. H. Townes received the Nobel Prize in Physics in 1964.

Magnetic semiconductors. Negative refractive index

After graduation, I naturally migrated into a researcher post in Prokhorov's group at FIAN and worked with him until his death in 2002. Topics of my research were very diverse and changed approximately every 5-7 years. In particular, since 1960 I was engaged in the research in the generation and use of strong dc magnetic fields. The magnetism subject led me to the study of so-called magnetic semiconductors. These substances, which are in some sense a "hybrid" of the magnetic and semiconductor materials, had a number of unusual properties. In particular, we hoped they could help us slow down electromagnetic waves. As known, the wave velocity depends on the refractive index n, which in turn is related to the dielectric permittivity ε and the magnetic permeability μ

$$n = \sqrt{\varepsilon\mu} \tag{1}$$

I wanted to obtain significantly higher values of n by increasing both ε and μ in the magnetic semiconductors. This would enable much lower velocity v of the wave:

$$v = \frac{c}{n} \tag{2}$$

where c is the speed of light in vacuum.

However, my efforts faced a major difficulty - the high values of ε and μ could not be realised simultaneously at any frequency. Moreover, often one of these quantities, ε or μ, became negative that led to an imaginary value of n, and thus the wave could not propagate in such a medium.

One day I looked incidentally at eq.(1), scribbled on a piece of paper, and suddenly asked myself a weird question: "What happens if both ε and μ of a medium will be negative?" Indeed, in accordance with eq. (1) the expression for n does not change! Does this mean that the equations of electrodynamics remain invariant with respect to the simultaneous change of the ε and μ signs? Or the properties of the media with negative ε and μ differ qualitatively from those of the media with positive ε and μ? Or is the medium with simultaneously negative ε and μ forbidden in principle because it might violate the basic laws of physics? I could not find instant answers to these questions, but I quickly recognised that these issues are of fundamental nature and we could not get away from them with the jokes like "it's all nonsense, it's purely mathematical trick, and there is no substance behind". Although many my colleagues recommended me to exercise a “take it easy” attitude to these niggling questions, I believed that the physics of media with simultaneously negative values of ε and μ contains something very important, and the recent developments give clear evidence that my intuition was right.

Then I locked myself in the office for several days and asked a secretary not to disturb me with phone calls telling everyone that I was away to unknown destination. During this time I wrote a paper[1] in which it was shown that if both ε and μ are negative, the minus sign should be ascribed to the phase velocity $\boldsymbol{v}$ and wave vector $\mathbf{k} = \omega/\mathbf{v} = n\omega/\mathbf{c}$, where ω - the wave frequency and the refractive index $n = c/v$.

Negative phase velocity $\boldsymbol{v}$ means that the wave vector is pointed oppositely to the Poynting vector $\boldsymbol{S} = [\boldsymbol{E} \times \boldsymbol{H}]$ whose direction from the source to the receiver is always positive. The possibility of the oppositely directed vectors $\boldsymbol{S}$ and $\boldsymbol{k}$ was mentioned earlier[2-5], but the notions of a negative refraction and negative refractive index n < 0 were not defined yet.

The very recognition of the fact that the wave vector $\mathbf{k}$ may be negative in the case of negative ε and μ implies the existence of the negative impulse $\boldsymbol{p}$, which in accordance with the principle of wave-particle duality is determined by the relation

$$\boldsymbol{p} = h\boldsymbol{k} \tag{3}$$

The negative impulse of the wave with negative $\boldsymbol{k}$ also means that if this wave is absorbed or reflected by a body, a pressure on the body is directed towards the radiation source but not out from it, as it is conventionally in the case of positive n

$$\boldsymbol{p} = n\omega/\boldsymbol{c} > 0 \tag{4}$$

Thus it appears that in the media with negative refractive index $n < 0$, the light pressure is replaced by the light attraction!

Negative refraction and some relativistic effects

This paradox with the light pressure was not the only new effect associated with the negative refraction. The latter phenomenon entailed several more unconventional effects. For example, it is worthwhile to examine the mass transfer from the source to the receiver, if it is carried out in a medium with negative refraction. Let us first consider such a mass transfer, following Einstein's paper[6].

To begin with, we assume that a radiator and receiver are placed in a vacuum at distance L from each other as shown in Fig. 1. A short pulse of light (wave packet or photon) is emitted by the radiator and reaches the receiver after time t

$$t = L/c \tag{5}$$

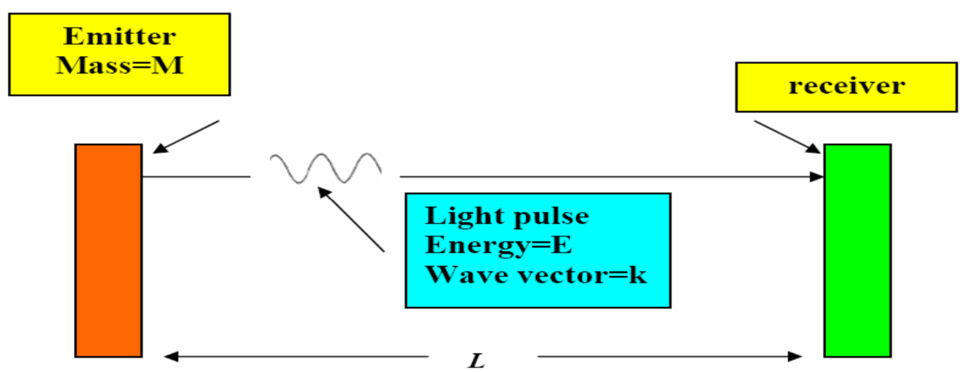

Figure 1

At the instance when the wave packet with energy E is emitted, the radiator acquires a recoil impulse

$$P = E/c \tag{6}$$

and thus has the speed

$$V = P/M \tag{7}$$

where M - the emitter mass. Then over the time when the light packet travels from the emitter to the receiver, the radiator moves to the left for a distance

$$x = Vt = LP/cM \tag{8}$$

Taking into account (6) we obtain from (8) that

$$xM = LE/c^2 \tag{9}$$

From this relation it is evident that for keeping the position of the centre of mass of the whole system unchanged, it is necessary that the radiative transfer of energy E from the radiator to the receiver be accompanied by the mass transfer to the same distance and

$$m = E/c^2 \tag{10}$$

At the same time, these photons, as known, have no mass at all.

The next nontrivial question is concerned with c^2 in(10). Indeed, what is the quantity c^2 and what is its real physical content? Does this quantity have a definite physical meaning, or it serves only to equate the dimensions at the right and left sides of eq. (10)? Finally, do the two c's in eq. (10) have the same or a different physical meaning? To clarify these questions, it is necessary to trace how these c's enter in eq. (10). It is easy to observe that one of them came from eq. (5) and corresponds to the group velocity c_{gr}, and the other one originates from eq. (6) and represents the phase velocity c_{ph}. Then the relation (10) can be rewritten as follows

$$m = E/\left(c_{gr} c_{ph}\right) \tag{11}$$

Thus it is also evident that if the whole space between the radiator and receiver is filled by the media where the waves travel with the phase velocity $\boldsymbol{v}_{ph}$ and group velocity $\boldsymbol{v}_{gr}$, the expression (11) takes the form

$$m = E/\left(\boldsymbol{v}_{gr} \cdot \boldsymbol{v}_{ph}\right) \tag{12}$$

From this relation it follows directly that when electromagnetic waves travels in the medium with negative refractive index and the velocities $\boldsymbol{v}_{ph}$ and $\boldsymbol{v}_{gr}$ are antiparallel, mass is transferred not from radiator to receiver but in the opposite direction, viz. from receiver to radiator. Hence, the classical eq. (10) represents a particular case of the more general relationship(12). Thus we can see that the concept of negative refraction and extension of the fundamental physical quantity of refractive index to include its negative values enables us to predict the new physical effects and significantly broaden the framework of the classical electromagnetic theory.

Conclusion

Unfortunately, experimental works on the negative refraction have started long after the appearance of theoretical studies[1-5]. The main reason for this delay was the lack of appropriate natural materials, in particular, with $n < 0$. The situation has changed dramatically after appearance of the work[7] which reinvigorated this exciting field of research and stimulated an explosive growth still going on at a high pace….

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