Abstract: Transformation electrostatics is a d.c. reduction of the transformation electromagnetic or transformation optics, which provides a freedom way to control the electric currents and potentials. Transformation optics/electromagnetics has been used to design and create a lot of novel devices theoretically and numerically, but few of them have been realized due to the difficulties in fabrications of inhomogeneous and anisotropic permittivity and permeability. However, transformation statics provides us an efficient methodology to design and realize novel d.c. devices freely. Using the analogy between electrically conducting materials and resistor networks, such d.c. devices can be designed using the circuit theory. We have fabricated a series of d.c. invisibility cloaks, exterior cloaks, active cloaks, concentrators, carpet cloaks, and illusion devices, etc. The measurement results agree exceptionally well with theoretical predictions and simulation results, showing perfect performance. Manipulation of steady currents with the control of anisotropic conductivities has a lot of potential applications, such as electric impedance tomography, graphene, natural resource exploration, and military hiding.

Keywords: Transformation electrostatics, conducting medium, resistor network, dc cloak, dc transformation devices, control of dc current.


Contents

- Brief Review
- Motivation of This Work
- Transformation Electrostatics Devices
- Conclusions
Transformation Optics

\[ \varepsilon'_u = \varepsilon_u \frac{Q_u Q_v Q_w}{Q^2_u}, \]

\[ \mu'_u = \mu_u \frac{Q_u Q_v Q_w}{Q^2_u}, \text{ etc.} \]
Comparing with the large amounts of theoretical work, only limited experiments have been conducted for transformation optics.

dc Transformation Optics

In the past a few years, the dc electric cloaks and magnetic cloaks have been theoretically proposed using the non-uniqueness property of anisotropic conducting materials and optical transformation.

Two Experiments Based on Superconducting Materials:


Following the idea by Wood & Pendry
Fig. 1. Calculated field lines for (A) a single cylindrical magnetic shell with $\mu = 3.54$, attracting fields and having some field penetration in its interior; (B) a single cylindrical superconducting shell with $\mu = 0$ repelling field lines; and (C) a cylindrical bilayer with an inner superconducting layer ($\mu = 0$) of interior (exterior) radius of $R_0 = 0.96$ $R_1$ ($R_1$) and an outer magnetic layer with $R_2/R_1 = 1.34$ with $\mu_2 = 3.54$, fulfilling Eq. 1. These values are chosen to approximate those used in the experiments. Green dotted lines denote the measuring lines in the experiments.
Our Motivation

To propose a general method to design and realize perfectly d.c. electric cloak;

To propose, design, and realize a series of transformation electrostatic devices.
**Perfect dc Electric Cloak**

A much cheaper cloak with perfect cloaking effect

Transformation Electrostatics

Theoretical Background

- Fundamental equations for steady electric fields

\[ \nabla \cdot \mathbf{J} = 0 \quad \nabla \times \mathbf{E} = 0 \quad \mathbf{J} = \sigma \mathbf{E} \]

- These equations look similar to those for electrostatics

\[ \nabla \cdot \mathbf{D} = 0 \quad \nabla \times \mathbf{E} = 0 \quad \mathbf{D} = \varepsilon \mathbf{E} \]

- They are form invariant under general coordinate transformation. Hence the transformation optics is valid.
Theoretical background

\[ \det(\mathbf{A}) = \mathbf{A}^\top \mathbf{A} \]

**Transformation Electrostatics**

<table>
<thead>
<tr>
<th>Physical Space</th>
<th>Virtual Space</th>
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<tbody>
<tr>
<td>( r_2(\varphi) )</td>
<td>( \mathbf{O} )</td>
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<tr>
<td>( r_1(\varphi) )</td>
<td>( \mathbf{O} )</td>
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\[
\tilde{\sigma}' = \frac{\mathbf{A} \tilde{\sigma} \mathbf{A}^\top}{\det(\mathbf{A})}
\]
dc Electric Cloak

(a) No objects;
(b) Uncovered object;
(c) Covered object;
(d) 2D case.

The conductivity profile contains zero point and singular value.

\[ \rho' = f(\rho) = \frac{b - a}{b} \rho + a, \quad \phi' = \phi, \quad z' = z, \]

\[ \tilde{\sigma}' = \Lambda \left[ \frac{\rho' - a}{\rho'}, \frac{\rho'}{\rho' - a}, \frac{\rho' - a}{\rho'} \left( \frac{b}{b - a} \right)^2 \right] \tilde{\sigma}. \]
Emulation of Conductivity Tensor with Resistor Network

A bulk conducting medium

\[ R_x = \frac{L}{\sigma_x \cdot S} = \frac{dx}{\sigma_x \cdot dy \cdot dz} \]

A periodic resistor network
2D: Rectangular Grids

\[ R_x = \frac{L}{\sigma_x \cdot S} = \frac{dx}{\sigma_x \cdot dy \cdot h} \]

\[ R_y = \frac{L}{\sigma_y \cdot S} = \frac{dy}{\sigma_y \cdot dx \cdot h} \]
2D: Cylindrical Grids

\[ R_\rho = \frac{\Delta \rho}{\sigma \rho \Delta \phi h} \]

\[ R_\phi = \frac{\rho \Delta \phi}{\sigma \Delta \rho h} \]
In the dc case, both zero point and singular value of conductivity have real physics meaning, and hence perfect cloaking effect can be realized.
Cloaking Performance

The central area is a perfect conductor.

The isotropic and homogeneous background material.
Cloaking Performance

The central region is cloaked.

Perfect cloaking performance is observed in measurements.
Nearly Perfect Cloak without Singularity

Jiang, et al., *APL* 93, 194102 (2008)
Further Work on dc Cloak

\[ \xi' = (\xi_2 - \xi_1) \xi / \xi_2, \quad \eta' = \eta \]

\[ \sigma_{\xi'} = \frac{\xi_2 - \xi_1}{\xi_2} \sigma_0 \]

\[ \sigma_{\eta'} = \frac{\xi_2}{\xi_2 - \xi_1} \sigma_0 \]

Homogeneous and Anisotropic; Perfect cloaking effect
Ultrathin but Nearly Perfect!

Jiang et al., *APL* 102, 014102 (2013)
Researchers have created a dc invisibility cloak made of a metamaterial that not only shields an object almost perfectly, but at 1-cm thick is also the thinnest cloak ever constructed, reaching the ultimate limit of thinness for artificial materials. As the first invisibility cloak that combines both near-perfect performance and extreme thinness, it could open the doors to practical applications. In the past, invisibility cloaks have been too large to be used in many real-world applications.

Brilliant.org Summer Camp - Developing tomorrow’s leaders in science, technology, and math. - brilliant.org/camp/

The researchers, led by Tie Jun Cui at Southeast University in Nanjing, China, have published their paper on the ultrathin but nearly perfect invisibility cloak in a recent issue of Applied Physics Letters.

The key to making a material that can prevent another object from being seen—or from being detected by electromagnetic waves in any way—is to control two material parameters: electric permittivity and magnetic permeability. Electric permittivity corresponds to the degree to which a material permits the formation of an electric field within itself, while magnetic permeability corresponds to the degree to which a material can be permeated by a magnetic field.

As the researchers explain, a perfect invisibility cloak must have a permittivity and permeability that are both strongly anisotropic (directionally dependent) and inhomogeneous (made of different materials). A metamaterial with these parameters is currently beyond the reach of current technology. However, by loosening these strict requirements, researchers have been able to fabricate metamaterials that mimic these properties and can be used as imperfect invisibility cloaks.
dc Illusion Device

Liu et al., APL 101, 051905 (2012).
(b) \[ \rho' = \frac{b - a}{b} \rho + a, \quad \varphi' = \varphi, \quad z' = z, \]

(c) \[ \bar{\sigma}' = \Lambda \left[ \frac{b - a}{b} \frac{\rho}{\rho'}, \frac{b}{b - a} \frac{\rho'}{\rho}, \frac{b}{b - a} \frac{\rho}{\rho'} \right] \sigma. \]
dc Carpet Cloak

Mei et al., Optics Express 20, 25758-25765 (2012)
Arbitrarily-Shaped Concentrator

- The rectangular concentrator can be easily used in amplifying plane waves.

dc Concentrator
dc Concentrator
dc Exterior Electric Cloak

Lai et al., PRL 102, 093901, 2009

Exterior Cloak: Folding-operation transformation; Only theoretical work

We propose dc exterior cloak, and make realization using active devices.

Folding-Operation Transformation

The transformation results in negative conductivity profile in the dc exterior cloak, which can be realized by negative resistors.
dc Exterior Electric Cloak

Realization of dc Exterior Cloak

dc Exterior Electric Cloak
Active dc Electric Cloak

Experiments on Active Cloaking and Illusion for Laplace Equation

We present the first experiment on active cloaking and propose an active illusion for the Laplace equation. We surround the central region with controlled sources to protect it from outside detection. We show that by dynamically changing the controlled sources, the protected region can be cloaked or disguised as different objects (illusion).

Ma et al., PRL 111, 173901, 2013
Active dc Electric Cloak

Ma et al., *PRL* 111, 173901, 2013
Active dc Electric Cloak

Ma et al., *PRL* 111, 173901, 2013
Active dc Electric Cloak

The simulated (left) and measured (right) potential distributions for the cloaking case.
We propose a cheaper but efficient way to represent transformation optics in the dc limit.

All the TO devices proposed in theory in time-varying fields can be realized in their counterparts in the dc field using the resistor network.

The method can be extended to control the heat transfers, which is also governed by the Laplace equation.
Thank you!

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