PARALLELIZED MULTILEVEL CHARACTERISTIC BASIS FUNCTION METHOD (MLCBFM) COMBINED WITH ADAPTIVE CROSS APPROXIMATION (ACA) FOR THE ANALYSIS OF THE SCATTERING FROM ELECTRICALLY LARGE ROUGH SURFACES

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OUTLINE

- INTRODUCTION
  a. Motivation

- ROUGH SURFACE STATISTICAL PROPERTIES

- TAPERED PLANE WAVES

- CBFM

- MULTILEVEL CBFM

- ADAPTICE CROSS APPROXIMATION ALGORITHM

- CODE VALIDATION

- ROUGH SURFACES WITH THE MLCBFM

- CONCLUSIONS AND FUTURE DEVELOPMENTS
MOTIVATION (1)

- The **Method of Moments (MoM)** is a widely used algorithm for the electromagnetic analysis of tridimensional scattering as well as circuit problems.

- The MoM associated matrix grows rapidly as the problem electrical dimensions become large or if a finite mesh is used to model complex structures in order to guarantee a reasonable solution accuracy.

- The memory allocation and the CPU time increase as $O(N^2)$ and $O(N^3)$ if $N$ is the total number of unknowns. This makes difficult the solution of electrically large problems by using the MoM.
MOTIVATION (2)

- The main scope of this presentation is the scattering analysis of the perfectly conducting Rough Surfaces via the Method of Moments

- The electromagnetic modeling of rough surfaces often requires several hundreds thousands of low level basis functions (RWGs or Rooftops)

- Conventional iterative solvers, such as the BICGSTAB(I) \([R_1]\) are characterized by converge issues for plane waves shining the surface from low-grazing angles or are very timing consuming for multiple RHS problems as the mono-static RCS type of analysis

We have analyzed one realization of a rough surface by using a conventional iterative solver:

- **Surface Realization:**

- Iterative solver: BICGSTAB
- Surface size: $16.0\lambda \times 16.0\lambda$
- Frequency: 300.0 MHz
- Number of RWGs: 53133
- Number of excitations: 160
- Solution type of 1 incident angle: 640s (Incident angle $(\theta_i, \phi_i) = (70, 0)\text{deg}$)
- Mono-static RCS solution time: 95421s (26.5 h)
ROUGH SURFACE STATISTICAL PROPERTIES

- The analyzed rough surfaces are characterized by a Gaussian correlation function $C(x,y)$ and by a Gaussian power spectral density distribution $W(k_x,k_y)$ \[ R_2 \]

\[
\langle f(x_1,y_1), f(x_2,y_2) \rangle = h C(|x_1 - x_2|, |y_1 - y_2|)
\]

\[
C(x,y) = \exp\left(-\frac{(x^2 + y^2)}{l^2}\right)
\]

\[
W(k_x,k_y) = \left(\frac{lh}{2\sqrt{\pi}}\right)^2 \exp\left(-\frac{l^2(k_x^2 + k_y^2)}{4}\right)
\]

- $C(x,y)$: correlation function
- $W(k_x,k_y)$: spectral density distribution
- $l$: correlation length
- $h$: profile RMS
- $f(x,y)$: surface profile

In order to avoid the edge effects from the “finite” analyzed surface, the tapered plane wave concepts reported in [R3] has been implemented.

\[ E_i = \exp\left[ - j k \cdot r (1 + w) \right] \exp\left[ - (t_x + t_y) \right] \]

\[ t_x = \frac{r \cdot \hat{\theta}}{(g \cos \theta_i)^2} \quad t_y = \frac{r \cdot \hat{\phi}}{g^2} \]

\[ w = \frac{1}{k^2} \left[ \frac{2t_x - 1}{(g \cos \theta_i)^2} + \frac{2t_y - 1}{g^2} \right] \]

- The tapering parameter \( g \) must be chosen according to the rough surface length \( L \)
- Small \( g \) values will produce inaccurate results relative to the specular reflection direction
- For all the results shown in this presentation we have chosen \( g = L/4 \)

CHARACTERISTIC BASIS FUNCTION METHOD

**Step - I:** The first step in CBFM is to divide the problem in smaller regions with respect to the original geometry, called “blocks” [R₄]

- The induced currents in each blocks are represented by using a type of high-level basis function defined **Primary Basis Functions**

**Step - II:** The **Primary Basis Functions** are evaluated by analyzing each block in the absence of the others:

\[
Z_{ii} \cdot P_{i,n} = V_{i,n} \quad i = 1,2,\ldots,M \\
n = 1,2,\ldots,N
\]

CHARACTERISTIC BASIS FUNCTION METHOD

- Each block is excited by using a Spectrum Of Plane Waves (SPWs) intentionally overestimating the Degrees Of Freedoms (DOFs) of the problem.

**Step - III:** After generating the CBFs, a thresholding procedure based on the a Singular Value Decomposition (SVD) approach, can be employed in order to reduce the number of CBFs and the reduced matrix size.

**Step - IV:** The final current distribution $J$ induced on the scatterer can be expressed as a linear combination of the CBFs provided by the SVD procedure ($J_{n,m}$):

$$J = \sum_{n=1}^{N_1} \alpha_{1,n} J_{1,n} + \sum_{n=1}^{N_2} \alpha_{2,n} J_{2,n} + \cdots + \sum_{n=1}^{N_M} \alpha_{M,n} J_{M,n}$$
CHARACTERISTIC BASIS FUNCTION METHOD

**Step – V:** The final step to be performed is the generation of the **Reduced matrix** $Z^r$ that can be accomplished by applying the Galerkin testing procedure employing the CBFs as testing functions

$$Z^r \cdot \alpha = V^r = J^T \cdot Z \cdot J \quad V^r = J^T \cdot V$$

$Z$: MoM impedance matrix  
$J$: matrix comprising all the CBFs after the SVD procedure  
$V$: RHS vector related to the real problem excitations

- The CBFM leads to a reduced matrix, which is much smaller than the original one, and this obviates the need for an iterative solution of problems requiring a large number of unknowns
MULTILEVEL CBFM

- In this paper a **Multilevel CBFM (MCBFM)** has been implemented in order to efficiently solve MoM problems characterized by more than 100,000 low-level basis function \([R_5]\)

- The compression rate between low-level and high-level basis functions increases as the size of the block and, hence, the number of basis functions inside it, is increased.

- The CBFs at the first level are constructed by using a spectrum of plane waves with a wide range of incident angles.

**References:**

MULTILEVEL CBFM

To determine the higher level CBFs, the lower level blocks are hierarchically grouped together

\[
Z^{(q)}_{\equiv i,j} = \begin{pmatrix}
Z^{(q-1)}_{\equiv 1,1} & Z^{(q-1)}_{\equiv 1,2} & \cdots & Z^{(q-1)}_{\equiv 1,K_i^{(q-1)}} \\
Z^{(q-1)}_{\equiv 2,1} & Z^{(q-1)}_{\equiv 2,2} & \cdots & Z^{(q-1)}_{\equiv 2,K_i^{(q-1)}} \\
\vdots & \vdots & \ddots & \vdots \\
Z^{(q-1)}_{\equiv K_i^{(q-1)},1} & Z^{(q-1)}_{\equiv K_i^{(q-1)},2} & \cdots & Z^{(q-1)}_{\equiv K_i^{(q-1)},K_i^{(q-1)}}
\end{pmatrix}
\]

\[
Z^{(q)}_{\equiv i,j} = \left(\left(J^{(q-1)}_{\equiv i,i}\right)^t Z^{(q-1)}_{\equiv i,j} J^{(q-1)}_{\equiv i,i}\right)
\]

Once the higher level expansion coefficients have been determined, the final current distribution can be uncompressed in a recursive manner

\[
J^{(r)} = \sum_{n=0}^{N-1} \alpha_n^{(Q-1)} J^{(Q-1)}_{n}(r) \\
J^{(q)}_{n,m}(r) = \sum_{p=0}^{N^{(q-1)}_n} \sum_{l=0}^{K^{(q-1)}_{n,p}} \alpha_l^{(q-1)} J^{(q-1)}_{p,l}(r)
\]
THE ADAPTIVE CROSS SECTION ALGORITHM

- The **Adaptive Cross Section Algorithm (ACA)** has been implemented in order to efficiently calculate the reduced matrix at each level of the proposed algorithm \([R_6]\).

- For well-separated groups of RWGs, the electric field at the observation group produced by any source RWG can be expressed as a linear combination of the fields produced by only a few of these source RWGs:

\[
Z^r_{i,j} = \left\langle J^t_{ii} Z^{RWG}_{i,j} J_{jj} \right\rangle \approx \left\langle J^t_{ii} \tilde{Z}^{RWG}_{i,j} J_{jj} \right\rangle
\]

\[
\tilde{Z}^{RWG}_{i,j} = U_{K_p \times r} \cdot V_{r \times K_p} \cdot Z_{i,i} - \tilde{Z}^{RWG}_{i,j}
\]

- The ACA algorithm is purely algebraic in nature and can be used irrespectively of the kernel of the integral equation, basis functions or type of integral equation formulation.

\[
\|R\|_F = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} |R_{i,j}|^2
\]

\[
\|R\|_F = \left\| Z^{RWG}_{i,j} - \tilde{Z}^{RWG}_{i,j} \right\|_F
\]

\[
\|R\|_F \leq \kappa \left\| Z^{RWG}_{i,j} \right\|_F
\]

\[
\|R\|_F = \sqrt{\sum_{n=0}^{N-1} \sum_{m=0}^{M-1} |R_{i,j}|^2}
\]

CODE VALIDATION

- The developed code has been validated by analyzing the canonical example of a Perfect Electric Conductor (PEC) sphere.

- The results provided by the MCBFM have been compared with the conventional CBFM ones and with the Mie solution.

- The code accuracy have been tested by using the reported Relative Error definition.

- Sphere radius: 3.0m
- Frequency: 300MHz
- Number of RWGs: 52,023
- SVD threshold: 1e^{-3}
- ACA threshold: 1e^{-3}
- Number of levels in the MCBFM: 2

\[
\text{Rel. Error} = 100 \sum_n \frac{|RCS_{MCBFM,n} - RCS_{Mie,n}|}{|RCS_{Mie,n}|}
\]
Very good agreement has been achieved between the proposed approaches.

The MLCBFM is characterized by a smaller reduced matrix which can be solved more efficiently. The reported time saving property is more evident when analyzing problems characterized by a larger number of low-level basis functions distributed in a small number of blocks.

<table>
<thead>
<tr>
<th></th>
<th>CBFM</th>
<th>MLCBFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blocks</td>
<td>64</td>
<td>16</td>
</tr>
<tr>
<td>Dimensions of the reduced matrix</td>
<td>8598</td>
<td>2932</td>
</tr>
<tr>
<td>Solution time(s)</td>
<td>551</td>
<td>171</td>
</tr>
</tbody>
</table>

Relative Error\((\%)\) = 0.3327
CODE VALIDATION

- We have also tested the numerical efficiency of the parallel algorithm by progressively increasing the number of CPUs employed to solve the sphere problem.
- The obtained efficiency has been compared with respect to the ideal case where the solution time scales as $1/n$.

The parallelized CBFM algorithm scales quite well.

CPU Time:

Parallel Implementation:

1. Multilevel Decomposition and Data Preprocessing
2. Generating CBFs for each block independently
3. Using MPI_Allgather to broadcast the CBFs
4. Generating the Reduced Matrix
5. Solve using LU Decomposition
ROUGH SURFACES WITH MLCBFM (1)

Surface Realization:

Monte Carlo approach:

\[ \eta_{RCS} = \frac{1}{N} \sum_{n=0}^{N-1} RCS_n \]

We have analyzed a rough surface profile, whose characteristics have been reported below, by using the CBFM and the MLCBM

- Frequency: 300MHz
- Rough surface dimensions (m): 16x16
- Number of RWGs: 53133
- Power spectral density type: Gaussian
- Profile RMS height \( h \): 0.2m
- Correlation length \( l \): 1.6m
- Incident angle \((\theta, \phi)\): \((70, 0)^\circ\)

Gaussian Power Spectral Density:

\[ W(k_x, k_y) = \left( \frac{lh}{2\sqrt{\pi}} \right)^2 \exp \left( - \frac{l^2(k_x^2 + k_y^2)}{4} \right) \]
ROUGH SURFACES WITH MLCBFM (1)

Bistatic RCS:

- Very good agreement has been achieved between CBFM and MLCBFM while the iterative solvers is characterized by......

Performance summary:

<table>
<thead>
<tr>
<th></th>
<th>CBFM</th>
<th>MLCBFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blocks</td>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>Dimensions of the reduced matrix</td>
<td>8832</td>
<td>3445</td>
</tr>
<tr>
<td>Solution time(s):</td>
<td>578</td>
<td>227</td>
</tr>
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</table>
ROUGH SURFACES WITH MLCBFM (1)

- We have calculated the RCS statistical distribution of the analyzed rough surface comparing the results with ones associated with its flat counterpart.

**y-z plane:**

![Graph of y-z plane](image1)

**x-z plane:**

![Graph of x-z plane](image2)
ROUGH SURFACES WITH MLCBFM (2)

- The CBF generation is time consuming and memory demanding since they need to be generated anew for each frequency [R7]
- Frequency sweep analysis can be made highly efficient by generating CBFs only for highest frequency of the band and used at lower frequencies too over 2:1 frequency band
- Scattering from a 8mx8m surface
- Frequency: 300MHz
- Frequency range (GHz): 0.3-0.6
- CBFs are generated at 0.6 GHz
- RCS is obtained at 0.3 GHz
- Incident angle ($\theta_i, \phi_i$): (70, 0°)

We have analyzed a rough surface profile, whose characteristics have been reported below, by using the CBFM, the MLCBM and the conventional PO.

- Frequency: 300MHz
- Rough surface dimensions (m): 32x32
- Number of RWGs: 194565
- Power spectral density type: Gaussian
- Profile RMS height $h$: 0.1m
- Correlation length $l$: 0.5m
- Incident angle $(\theta_i, \phi_i)$: $(80, 0)^\circ$

<table>
<thead>
<tr>
<th></th>
<th>CBFM</th>
<th>MLCB FM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blocks</td>
<td>64</td>
<td>8</td>
</tr>
<tr>
<td>Dimensions of the reduced matrix</td>
<td>18528</td>
<td>8284</td>
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<tr>
<td>Solution time(s)</td>
<td>1763</td>
<td>556</td>
</tr>
</tbody>
</table>
ROUGH SURFACES WITH MLCBFM (4)

- We have tested the possibility of neglecting EM interaction between blocks separated by an electrical distance greater than a certain threshold.
- Rough surface dimensions ($\lambda$): 4x4
- Separation distance ($\lambda$): 2
- Number of RWGs: x
- Power spectral density type: Gaussian
- Profile RMS height $h$: x
- Correlation length $l$: x
- Incident angle ($\theta_i$, $\phi_i$): (0, 0)°

Surface Realization:

x-z plane:

y-z plane:
ROUGH SURFACES WITH MLCBFM (4)

- We have calculated the RCS statistical distribution of the analyzed rough surface
- The CBFs have been generated at 0.6GHz
- The RCS distribution has been calculated at 0.3GHz
We have tested the possibility of neglecting EM interaction between blocks separated by an electrical distance greater than a certain threshold.

- Rough surface dimensions ($\lambda$): 16x16
- Number of RWGs: xxxxx
- Power spectral density type: Gaussian
- Profile RMS height $h$: xxxxx
- Correlation length $l$: xxxxx
- Incident angle ($\theta_i, \phi_i$): (0, 0)$^\circ$
- Separation distance threshold ($\lambda$): 4

**Surface Realization:**

**, Bistatic:**

**Monostatic:**
CONCLUSIONS AND FUTURE DEVELOPMENTS

- The Multilevel Characteristic Basis Function Method has been applied to analysis of the electromagnetic scattering from randomly generated rough surfaces.

- The MLCBFM is an iteration-free, high parallelizable approach which can efficiently solve scattering problems characterized by multiple Right Hand Sides and grazing incident angles.

- In order to reduce the computational burden, the CBFs have been generated only at the highest frequency of interest and the electromagnetic interaction between blocks separated by an electrical distance above a certain threshold has been neglected.

- Very good agreement has been achieved between the proposed MLCBFM and the conventional MoM, while the Physical Optic approach does not provide accurate results.

- As a future development, we are currently investigating an efficient way to calculate the CBF interaction in the wave-number domain.