Highly-Efficient Multi-Coil Wireless Power Transfer (WPT)

Mehdi Kiani

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GT-Bionics Lab, School of Electrical and Computer Engineering
Georgia Institute of Technology, Atlanta, GA
WPT Applications

Charging mobile electronics

Implantable Medical Devices (IMDs)

Charging electric cars

RFID
Coupled-mode Magnetic Resonance-based Power Transmission

• Proposed by physicists at MIT based on coupled-mode theory (CMT)
• An alternative wireless power-transfer technique using typically four coils
• Goal: Increase the power transfer efficiency (PTE) at large coupling distances

60-W is transferred from 2-m away
Questions?

• A need for a comprehensive circuit theory for coupled-mode magnetic resonance-based links
• A need for circuit theory based on the reflected-load theory (RLT) for multi-coil links
• Are coupled-mode magnetic resonance-based links always the optimal choice?
• A need for a new figure-of-merit (FoM) for high performance multi-coil design

![Resonant-magnetic coupling vs. Inductive coupling](image)
Calculating Power Transfer Efficiency (PTE) based on RLT

- Secondary $R_3L_3C_3$ can be reflected onto the primary:

$$PTE = \frac{R_{ref}}{R_2 + R_{ref}} \frac{Q_3^2R_3}{Q_3^2R_3 + R_L} = \frac{k_{23}^2Q_2Q_{3L}}{1 + k_{23}^2Q_2Q_{3L}} \cdot \frac{Q_{3L}}{Q_L}$$

- PTE is highly dependent on: $k_{23}$, $Q_2$, and $Q_3$

$$Q_{3L} = Q_3Q_L / (Q_3 + Q_L)$$

$$Q_L = R_L / \omega_0 L$$

\[ \begin{align*}
R_{ref} &= k_{23}^2 \left( \frac{L_2}{L_3} \right) (Q_3^2R_3 \parallel R_L) = k_{23}^2 \omega_0 L_2 Q_{3L} \\
C_{ref} &= \left( \frac{L_3}{L_2} \right) (C_3 / k_{23}^2) = 1 / (\omega_0^2 L_2 k_{23}^2)
\end{align*} \]
Maximizing PTE in 2-Coil Link

\[ \eta_{2-coil} = \frac{k_{23}^2 Q_2 Q_{3L}}{1 + k_{23}^2 Q_2 Q_{3L}} \cdot \frac{Q_{3L}}{Q_L} \]

- For a given set of \( Q_2, Q_3 \) and \( k_{23} \) values, there is an optimal load, \( R_{L,PTE} \), which can maximize the PTE at that particular arrangement, such as the coupling distance.

\[ R_{L,PTE} = \omega_0 L_3 Q_{L,PTE} \]
\[ Q_{L,PTE} = \frac{Q_3}{(1 + k_{23}^2 Q_2 Q_3)^{1/2}} \]

- PTE is highly dependent on \( R_L \)
- \( R_L \) is often predefined by the application

**Impedance Transformation**

1) Matching circuits
2) **Multiple coils**
PTE and Delivered Power (PDL) in Multi-coil Inductive Links based on RLT

- In an \( m \)-coil link with negligible coupling between non-neighboring coils
  - \( R_{ref} \) from \((i+1)\)th coil to the \( i \)th coil: \( R_{ref\ i,i+1} = k_{i,i+1}^2 \omega_0 L_i Q_{(i+1)L} \)
  - Loaded \( Q \) of the \( i \)th coil: \( Q_{iL} = \omega_0 L_i / (R_i + R_{ref\ i,i+1}) \)
  - PTE between \( i \)th and \((i+1)\)th coils: \( \eta_{i,i+1} = R_{ref\ i,i+1} / (R_i + R_{ref\ i,i+1}) \)
  - \( m \)-coil link PTE: \( \eta_{m-coil} = \prod_{i=1}^{m-1} \eta_{i,i+1} \cdot Q_{mL} / Q_L \)
  - PDL: \( P_{L,m-coil} = \frac{V_s^2}{2R_1} \frac{1}{1 + k_{12}^2 Q_1 Q_2 L} \eta_{m-coil} \)

- Parallel load: \( Q_{mL} = R_L / \omega_0 L_m \)  
- Series load: \( Q_{mL} = \omega_0 L_m / R_L \)
PTE in Multi-coil Inductive Links based on CMT

• In \( m \)-capacitively loaded resonators:

\[
\begin{align*}
\frac{da_1(t)}{dt} &= -(j\omega + \Gamma_1)a_1(t) + jK_{12}a_2(t) + F_S(t) \\
\frac{da_i(t)}{dt} &= -(j\omega + \Gamma_i)a_i(t) + jK_{i-1,i}a_{i-1}(t) + jK_{i,i+1}a_{i+1}(t) \\
\frac{da_m(t)}{dt} &= -(j\omega + \Gamma_m + \Gamma_L)a_m(t) + jK_{m-1,m}a_{m-1}(t)
\end{align*}
\]

\[
\eta_{m\text{-coil}} = \frac{P_L}{P_S} = \frac{\Gamma_L}{\Gamma_m + \Gamma_L + \sum_{i=1}^{m-1} \left| \frac{A_i}{A_m} \right|^2}
\]

Resonance width: \( \Gamma_i = \omega_0 / 2Q_i \)

Coupling rate: \( K_{i,i+1} = \omega_0 k_{i,i+1} / 2 \)

\[
a_i(t) = A_i e^{-j\omega_0 t}
\]

\[
P_i = 2\Gamma_i |A_i|^2
\]

Four-coil link (\( m = 4 \))

PTE from CMT is identical to the one from RLT!

M. Kiani and M. Ghovanloo, TCAS-I 2012
Resonant Magnetic Coupling vs. Inductive Coupling (Transient)

1) Mid-range high-Q condition

2) Short-range low-Q condition

CMT transient response is accurate when coils coupling is weak and coils quality factor is very high!

M. Kiani and M. Ghovanloo, TCAS-I 2012
3-Coil Inductive Link

\[ \eta_{3\text{-coil}} = \eta_{23} \eta_{34} = \frac{(k_{23}^2 Q_2 Q_3)(k_{34}^2 Q_3 Q_{4L})}{[(1 + k_{23}^2 Q_2 Q_3 + k_{34}^2 Q_3 Q_{4L})(1 + k_{34}^2 Q_3 Q_{4L})]} \cdot \frac{Q_{4L}}{Q_L} \]

- L₃-L₄ inductive link provides designers with a DoF \( M_{34} \) to adjust the reflected load on to \( L_3 \) to be the optimal value: \( R_{L,PTE} \)
- Loosely coupled L₂-L₃ link is designed for optimal \( R_{L,PTE} \)
- Inductive link optimization is decoupled from \( R_L \)
- L₃-L₄ inductive link PTE is high as their coupling distance is small

\[ R_{L,PTE} = \omega_0 L_3 Q_{L,PTE} \quad Q_{L,PTE} = \frac{Q_3}{(1 + k_{23}^2 Q_2 Q_3)^{1/2}} \]
Maximizing PTE in the 3-Coil Link

By changing $k_{34}$ (and $R_{ref,3}$), the 3-coil PTE can be kept at maximum for a wide range of $R_L$.

A 2-coil link does not provide this flexibility, and PTE maximizes only for a specific $R_L$ value.

M. Kiani and M. Ghovanloo, TBCAS 2011
**4-Coil Inductive Link**

\[ \eta_{4\text{-coil}} = \eta_{12} \cdot \eta_{23} \cdot \eta_{34} = \frac{(k_{12}^2 Q_1 Q_2)(k_{23}^2 Q_2 Q_3)(k_{34}^2 Q_3 Q_4)}{[(1 + k_{12}^2 Q_1 Q_2)(1 + k_{34}^2 Q_3 Q_4 L) + k_{23}^2 Q_2 Q_3] [1 + k_{23}^2 Q_2 Q_3 + k_{34}^2 Q_3 Q_4 L]} \cdot \frac{Q_{4L}}{Q_L} \]

- 4-Coil link adds an additional DoF for impedance matching on the source side
- If \( k_{12} \) is large, the reflected load onto \( L_1 \) increases dramatically, which helps maximize the PTE at the cost of reducing PDL
Maximizing PTE should not be at the cost of decreasing PDL

\[ P_{L,3-coil} = \frac{V_s^2}{2 R_2} \frac{(k_{23}^2 Q_2 Q_3)(k_{34}^2 Q_3 Q_{4L})}{(1 + k_{23}^2 Q_2 Q_3 + k_{34}^2 Q_3 Q_{4L})^2} \cdot \frac{Q_{4L}}{Q_L} \]

The optimal design maximizes both PTE and PDL

\[ k_{23,PDL} = \left( \frac{1 + k_{34}^2 Q_3 Q_{4L}}{Q_2 Q_3} \right)^{1/2} \]
\[ k_{34,PDL} = \left( \frac{1 + k_{23}^2 Q_2 Q_3}{Q_3 Q_{4L}} \right)^{1/2} \]
PTE and PDL in 4-Coil Link

- If $k_{12}$ is large enough, 4-coil can tolerate variations in coil separation ($k_{23}$) and maintain a large PTE.
- Large $k_{12}$ reduces the available power from the source.
- Small overlap between high PTE and PDL areas.

M. Kiani and M. Ghovanloo, TBCAS 2011
3-Coil vs. 4-Coil Link (Measurements)

V_s = 1 V

3-coil link (PDL = 260 mW)

4-coil link (PDL = 4.4 mW)

M. Kiani and M. Ghovanloo, TBCAS 2011
**Resonant-Magnetic Coupling vs. Inductive Coupling (PTE)**

Calculated PTEs were similar from CMT and RLT and matched very well with measurements!

Four-coil link measurement setup. Three and two-coil links used similar coils except $L_1$ and $L_4$, respectively.

M. Kiani and M. Ghovanloo, *TCAS-I* 2012
A New Figure of Merit (FoM) for Inductive Power Transmission

- Conventional design merits are either PTE or PDL
- How to balance PTE and PDL?
- How to choose between 2-, 3-, and 4-coil links?

\[
FoM = \frac{\eta_{m-coil}^n \times P_{L,m-coil}}{V_s^2}
\]

\(n\): PTE weight, which depends on the application

- FoM effect on PTE and PDL drop:

\[
\eta_{Loss} = \frac{\eta_{max} - \eta_{FoM}}{\eta_{max}} = \frac{1}{n+2},
\]

\(\eta_{max}, P_{L,max}\): PTE and PDL are maximized

\[
P_{L,loss} = \frac{P_{L,max} - P_{L,FoM}}{P_{L,max}} = \frac{n^2}{(n+2)^2}
\]

\(\eta_{FoM}, P_{L,FoM}\): FoM is maximized

M. Kiani and M. Ghovanloo, TIE 2013
PTE vs. PDL Loss based on FoM for 2-, 3-, and 4-coil Links

FoM = \( \eta_{m-coil}^n \times P_{L,m-coil} \div V_s^2 \)

\( n = 0 \) \( \rightarrow \) FoM ~ PDL

\( n \rightarrow \infty \) \( \rightarrow \) FoM ~ PTE

\( n = 2 \) results in similar PTE and PDL drops = 25%

M. Kiani and M. Ghovanloo, TIE 2013
Optimal 2-coil Link for IMDs based on FoM

- 2-coil Link for IMDs
- Rx coil dia. = 10 mm
- $V_s = 1$ V
- $R_s = 0.5$ Ω
- $R_L = 100$ Ω
- $f_0 = 13.56$ MHz
- $d_{23} = 10$ mm

At $d_{23} = 10$ mm, FoM-optimized link provides 47% more PTE than the PDL-optimized link and 16 times larger PDL than the PTE-optimized link.
Comparing 2- 3- & 4-Coil Links for IMDs

- At $d_{23} = 10$ mm, 4-coil link provides 4.9% more PTE and 11 times less PDL than an equivalent 3-coil. At $d_{23} = 20$ mm, PTE difference is 7.7%, while PTE of 3-coil is 192 times larger than an equivalent 4-coil.
- For large $R_s$, 4-coil is optimal.

**An FoM including both PTE and PDL is needed to differentiate between 2-, 3-, and 4-coil links!**
Optimal Multi-coil Link for Charging Handheld Mobile Devices based on FoM

The 4-coil link has superior FoM at $d_{23} = 10$ cm at the cost of much lower PTE, and consequently the FoM, at shorter coupling distances.

Rx coil dia. = 4 cm, $R_s = 0.5 \, \Omega$, $R_L = 5 \, \Omega$, $f_0 = 13.56$ MHz, $d_{23} = 10$ cm

M. Kiani and M. Ghovanloo, TIE 2013
## Optimal Multi-coil Link based on FoM

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<thead>
<tr>
<th>Conditions</th>
<th>2-Coil Link</th>
<th>3-Coil Link</th>
<th>4-Coil Link</th>
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<tr>
<td>Strong coupling ($k$)</td>
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<tr>
<td>Weak coupling ($k$)</td>
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<td>Large PDL (small $R_s$)</td>
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<td>Small PDL (large $R_s$)</td>
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