Computer-aided Teaching of Electromagnetics: Is It for Real?

There is no denying of the fact that we live in a world in which the presence of the computers is ubiquitous and that they dominate our lives directly or indirectly every which way we turn. So, it is not surprising for us to pose the burning question: Should we be teaching Electromagnetics solely by using Maxwell’s equations, and those pertaining to waveguides, antennas, propagation, etc., derived from those celebrated equations, or should we bring computers into the picture so that the student develops a better understanding of the EM theory? EM is undoubtedly quite abstruse for most students, because of the complexities of vector calculus, integral and differential equations, algorithms for solving boundary value problems, and such.

In the face of noticeable decline in the number of students opting to choose EM over Communication, Signal Processing, Material Science and Nanotechnology, it would be useful for us to do a bit of self-examination, and soul-searching, to see what we can do to make EM a little more palatable and not so foreign to the twenty-something batch of students who are weaned on Facebook, Twitter, Google and similar sites that have undoubtedly shaped their minds, as Steve Jobs would say, “To Think Different” (pardon my grammar).

There is yet another cogent reason why we should leverage the power of the computer to teach EM theory to the students. At this point you must be wondering, dear reader, how can we utilize the computer, which is nothing but a number-cruncher, to teach theory to the students? I’ll answer this question with two practical examples, involving a rectangular waveguide and a microstrip patch antenna.

For Example-1, we refer to Slide-4, which presents the geometry of a waveguide excited with a probe feed. The student is asked to use a computer code, available off-the-shelf, to compute and plot the field distributions at various distances from the probe—near, intermediate and far. The objective of the exercise is to teach the student about the role played by the modes in representing the fields inside a waveguide; how these field behaviors can be interpreted in terms of propagating and evanescent modes; how to compute the excitation coefficients of these modes; how to predict the effect of a slight asymmetry of the probe location on the type of modes it excites; and so on. The numerical results shown in Slides 5-15 are used to discuss these issues.

Another very important fallout of this exercise is to learn how the theoretically derived result can be used to validate the numerical ones that are generated by the computer, since the results can wander all
over the map unless a correct mesh, proper discretization, and the right convergence criterion, etc., are used to compute the results. Yet another benefit of this exercise is that the student can learn how to express a given field distribution inside the waveguide in terms of a set of basis functions by deriving a matrix equation, say by using the Galerkin method, and also understand how this matrix equation becomes diagonalized when we choose the basis functions to be the modes of the waveguide that are orthogonal to each other.

I submit that a student who has simply gone through the material in a book chapter which discusses waveguides—and most EM books include such a chapter—seldom ever develops the level of comprehension of the physics of guided-wave propagation that is necessary to fully understand the field behavior in a guided-wave structure. He would acquire this understanding if he were to go through the analysis and interpretation of the field behavior, derived by simulating on the computer the problem of waveguide excitation by a probe-feed. He might also appreciate the difficulties associated with the application of the Green’s function approach to solving the field excitation problem by a “real” rather than idealized source, as for instance the one shown in Slide-4.

For the second example, we consider the problem of a patch antenna, fed by a microstrip line (see Slide-17). Let us say that the student is asked to compute $S_{11}$, or the return loss, from the field solution generated by using an FDTD code, which is provided to him. The microstrip line is also fed by a probe, say the center conductor of a coax located below the ground plane.

Suppose the student proceeds to use the knowledge of his transmission line theory to find the input impedance $Z_{in}$, by sampling the current at the feed, derived by using the H-field circumscribing the microstrip line ($\int \mathbf{H} \cdot d\mathbf{l}$), and calculating the voltage by integrating the $E$-field ($\int \mathbf{E} \cdot d\mathbf{l}$) from the ground to the microstrip line. He can then compute the $S_{11}$, using the $Z_{in}$ and the characteristic impedance $Z_o$ of the microstrip line, and derive the frequency domain results for $S_{11}$ by Fourier transforming the time domain field data.

While the procedure describes above is theoretically sound, the numerical result obtained by following this procedure is not always accurate, and may not satisfy the passivity condition. At this point, the student may turn to his class notes, or the prescribed textbook for an EM course, and search frantically through the book to figure out where he might have gone wrong. Unfortunately, to the best of my knowledge, there is no book that is going to come to his rescue, and the student would be left out in the cold, with no help in sight.

This is the point where a computer-aided approach to resolving this problem, and developing an understanding of its physics really saves the day.
If the student analyzes the current density distribution on the uniform piece of microstrip line, away from both the feed point, as well as the junction point between the line and the microstrip patch, then and only then he would find that the distribution can be represented by just a combination of forward and backward traveling waves associated with the dominant mode. Note that this would not be true if either of the two ends of the uniform line were included in the analysis of the current representation, and the derivation of the $S_{11}$-parameter was carried out by using this representation to compute the levels of forward-and backward-going waves in the microstrip line feeding the patch antenna. Three approaches of extracting this information, namely Prony, Pseudo-inverse and GA methods, are described in Slides 26-38.

Not only would the student be able to obtain the correct results from the approach described above where the two end points and their proximity regions are excluded from the analysis, but he would also gain some understanding of the effect of the presence of higher-order modes in microstrip lines, that are undoubtedly present in real-world problems. In addition, he would be exposed to the notion of de-embedding, which is always necessary to implement in order to obtain accurate results for problems of this type.

Finally, before closing, we refer to Slide-39, which shows two other examples where we would benefit by carrying out the simulation of the physical structure, computer-intensive as that exercise may be. In the past, many researchers have relied solely on effective medium theories to represent the hemispherical radome (Slide-39) or the flat lens (Slide-40), arguing that computer simulation of these problems is not only very expensive, but that such an exercise provides little physical insight; hence they should be avoided, they would say. However, the use of the effective medium approach can sometimes lead to misleading and inaccurate conclusions. For instance, one might think that a small antenna could be matched by using an EN (epsilon negative) radome, and that it would achieve a very high directivity despite its small size. It has also been argued that a DNG flat lens would behave as a superlens and would beat the diffraction limit. Unfortunately, such predictions were found to be very far removed from reality once the physical structures—and not the models based on effective medium theories—were simulated by using numerically rigorous algorithms. A few other examples that fall in this category are shown in Slides 41-55. Once again, for many such examples, modeling the physical structure ‘as is’ has been found to trump over the theoretical predictions based on over-simplified theories, e.g., those utilizing the effective medium approach. This would lead us to recommend that this approach, while it is simple, elegant and physics-based, should only be used within the range of its validity.
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Raj Mittra
Director, Electromagnetic Communication Lab
Penn State University
USA
COMPUTER-AIDED TEACHING OF EM
Problem#1
E-field Distribution in an X-band Waveguide
X-Band waveguide, frequency Band : 8.20–12.40 GHz
E field along line for $f=4\text{GHz}$
Ez field along different Observation Planes from source, for $f=4\text{GHz}$
Ez-Field distribution, xz cross-section, for $f=4\text{GHz}$

**Ez(x,z): Obs. Plane-1, $y=y_1$**

**Ez(x,z): Obs. Plane-2, $y=y_2$**

**Ez(x,z): Obs. Plane-4, $y=y_4$**
E field along line for $f=10$ GHz

Observation Line

Ez(x,z): Obs. Plane-4, $y=y_4$
Ez(x,z): Obs. Plane-3, $y=y_3$
Ez(x,z): Obs. Plane-2, $y=y_2$
Ez(x,z): Obs. Plane-1, $y=y_1$
Ez field along different Observation Planes from source, for $f=10\text{GHz}$
Ez-Field distribution, xz cross-section for $f=10\text{GHz}$
E field along line for f=14 GHz

Observation distance along y-axis in mm

Ez(x,z): Obs. Plane-1, y=y1
Ez(x,z): Obs. Plane-2, y=y2
Ez(x,z): Obs. Plane-3, y=y3
Ez(x,z): Obs. Plane-4, y=y4

Normalised E_z-Field

E-Field Distribution along line for f=14GHz in X-band Waveguide

Observation Line

E-field along line for f=14 GHz
Ez field along different Observation Planes from source, for $f=14\text{GHz}$.
Ez-Field distribution, xz cross-section for f=14GHz
From fig. 1, which is the Obs. plane-1, y=y1, near the coaxial probe, it is clear that higher order modes might be present near the probe location, due to which the Ez field distribution as a function of x-variation looks different.
From fig. 1., which is at Obs. Plane-4, y=y4, near the end of waveguide, the Ez field distribution as function of x-variation for different values of ‘z’ has same level, for which we can say that the mode is settled, which is $TE_{10}$ mode distribution.
PROBLEM#2
S-PRAMETER CALCULATION OF A MICROSTRIP PATCH ANTENNA
Analysis of Microstrip line (MS) and Line-fed Rectangular Microstrip Antenna and calculation of S-parameter ($S_{11}$)

- $f=10\text{GHz}$
- Parameters: $\Delta x=0.389\text{mm}$, $\Delta y=0.4\text{mm}$, $\Delta z=0.265\text{mm}$,
- Domain = $N_x=60+pml$, $N_y=100+pml$, $N_z=9+pml$,
- Line fed width = 6 cell,
- Line fed length = 50 cell,
- Time steps = $n_{max}= 200, 300, 500, 5000$ for checking the field entering from the i) strip line ii) line fed strip to the Microstrip patch antenna.
Source entering into the Microstrip-line at nmax=200,400

Amplitude of Ez along Y-Plane, Iteration = 200
Amplitude of Ez along Y-Plane, Iteration = 400

Amplitude in V/m

Ez at (npml+2 cell) along z-axis.
Pulse Propagation underneath the interface, nmax=500,

Amplitude of Ez along Y-Plane, Iteration = 500

Ez at (npml+2 cell) along z-axis.
Comparison of Time Signature’s of Microstrip line and line fed patch for 5000 timesteps, at 21\textsuperscript{th} cell location

![Graph showing Time Signature at 21 cell on Line fed Patch with Time Steps on x-axis and Amplitude on y-axis. The graph compares Time Signature on line fed patch (blue line) and Time Signature on MS line (black line).]
Time Signatures at 21\textsuperscript{st} cell location

![Diagram with 3D models and graphs showing time signatures at 21\textsuperscript{st} cell location on MS line and Line fed Patch.](image-url)
Transformation from time domain to frequency domain based on DFT in general is given as:

\[ G(x, y, z, f) = dt \sum_{n=0}^{N-1} g(x, y, z, ndt) \exp(-j2\pi fn dt) \]

Here \( g(x,y,z,ndt) \) is the pulse response, \( dt \) is the time interval and \( N \) being the number of samples. Here we have 5000 samples at 21 cell location as Pulse response from the FDTD for both cases of MS line and Line-fed rectangular patch antenna.
S_{11} calculation

Here \( DFT[v_{inc}(t)] \) is considered for MS line and \( DFT[v_{tot}(t)] \) for line fed Patch antenna, because \( v_{inc}(t) \) is taken as time signature extracted at particular cell location from Microstrip line, and \( v_{total}(t) \) is taken as time signature extracted at particular cell location from Line-fed patch antenna, but \( S_{11} \) is calculated as

\[
S_{11}(\omega) = \frac{DFT[v_{ref}(t)]}{DFT[v_{inc}(t)]}
\]

But here \( v_{ref}(t) = v_{tot}(t) - v_{inc}(t) \), so

\[
S_{11}(\omega) = \frac{DFT[v_{tot}(t) - v_{inc}(t)]}{DFT[v_{inc}(t)]}
\]

Furthermore, We calculate S11 from above equations and compare this value with other commercial solver(i.e. HFSS).
Comparison of S11, FDTD and HFSS, taking one specific cell location underneath the line, e.g., 2nd cell along z.
$S_{11}$ of Line fed Patch Using Prony, Pseudo-Inverse and GA method.
Pseudoinverse method.
f=10GHz
Parameters: $\Delta x=0.389\text{mm}$, $\Delta y=0.4\text{mm}$, $\Delta z=0.265\text{mm}$,
Domain $=N_x=60+pml$, $N_y=100+pml$, $N_z=9+pml$,
Line fed width = 6 cell,
Line fed length = 50 cell,
Time steps $=n_{max}=200$, 300, 500, 5000 for checking the field entering from the i) strip line ii) line fed strip to the Microstrip patch antenna.

Run the FDTD code for extracting enough samples, which are time domain data from actual geometry.

Samples from FDTD
(i.e., Time Domain data)
Taking DFT and converting all data in frequency domain for processing at individual frequencies

Samples from FDTD (i.e. Time Domain data)

Taking DFT
Taking absolute

Field at each sample, f=10GHz,
Here $f(x)$ is 44x1 matrix, C is 2x1 matrix, coefficients to be calculated from pinv method, E is 44x2 matrix, Exponents, where E1, E2 are fetched from GPOF method. Applying the Pseudoinverse method on this for finding the coefficients which are complex numbers.

Here following are the reasons to apply the Pseudoinverse method to solve and find the coefficients.

i) Here more equations then Unknowns
ii) No. of rows are more than no. of Columns
iii) No. of rows are more than no. of variables
i.e. system is over determined.
Moore Penrose Pseudoinverse (pinv), method

Using the above method we calculate the coefficients $C_{1rpinv}, C_{1ipinv}, C_{2rpinv}$ and $C_{2ipinv}$ then putting in the following equation to form the predicted function based on the coefficients calculated using p_inv method.

$$f_{pinv}'(x) = (C_{1rpinv} + C_{1ipinv} \times j)e^{-0.1185j\times x} + (C_{2rpinv} + C_{2ipinv} \times j)e^{+0.1185j\times x}$$

The above function $f_{pinv}'(x)$ is calculated for one frequency i.e $f=10$GHz, further in similar way we find the coefficients for other frequencies to calculate the $S_{11}$ parameters, which is shown in last slide for final comparison with other methods.
Genetic Algorithm (GA) method
For example, f=10 GHz, which is in form of complex number, saying this as $f(x)$, which is the measured data.
Using Genetic Algorithm to find coefficients i.e., $C_{1\text{gar}}$(real), $C_{1\text{gai}}$(imaginary), $C_{2\text{gar}}$(real) and $C_{2\text{gai}}$(imaginary) by minimization of Cost function (i.e. Error or residuals). Here exponents are taken from GPOF method as shown in previous slide.

The steps involved are shown in next slide to find cost function and then minimize it.
Cost function: ‘y’

\[ \delta(i) = f(x) - f_{ga}'(x); \quad \% \text{finding the Error i.e. difference of Complex numbers} \]
\[ y_1(i) = |\delta(i)|; \quad \% \text{absolute of Error} \]
\[ y = \text{sum}(y_1^2) \quad \% \text{Square of Error} \]

Here ‘y’ is the cost function, \( f(x) \) is the measured data, \( f_{ga}'(x) \), which is from GA optimization method.

Here GA method searches for the coefficients by minimizing the cost function, when threshold is reached, set by the algorithm it comes out from the algorithm and plots the \( f_{ga}'(x) \)

Finally comparing the measured function \( f(x) \) with fitted function \( f_{ga}'(x) \), for 10GHz is shown in next slide, where it is clear from the plots that algorithm goes under sufficient iterations to solve and find the coefficients.
Function values Vs Iterations using Genetic Algorithm for Lower Boundary (LB) and Upper Boundary (UB)

Coefficients from Genetic Algorithm

- \( C_1 = 0.00568107903160472 + 0.00288845677151099i \)
- \( C_2 = -0.000928150512839350 + 0.00197809744670331i \)

Coefficients from Pseudo Inverse

- \( C_1 = 0.00595680695992498 + 0.00218394569259418i \)
- \( C_2 = -0.00123618608213028 + 0.00181973928626468i \)
Normalized Plot

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Comparison of Input function and fitted data, 10GHz

Coefficients from Genetic Algorithm

\[ C_1 = 0.00568107903160472 + 0.00288845677151099i \]

\[ C_2 = -0.000928150512839350 + 0.00197809744670331i \]

Coefficients from Pseudo Inverse

\[ C_1 = 0.00595680695992498 + 0.00218394569259418i \]

\[ C_2 = -0.00123618608213028 + 0.00181973928626468i \]
The above plot is comparison of $S_{11}$ using different methods for Line-fed rectangular patch antenna, it clear from above plot, that $S_{11}$ calculated from different methods like pinv, and GA method are close to each other except some difference, comparing with other commercial software i.e HFSS.
SMALL ANTENNAS WITH VERY HIGH DIRECTIVITIES?
THE PERFECT LENS?
Negative Refraction and Lensing
Introduction to Metamaterials

- A Metamaterial (MTM) is a composite “artificial material” engineered to produce desired electromagnetic (EM) propagation behavior not found in natural materials.

- MTM EM propagation results from its structure rather than from the specific materials of which it is composed.

- MTM structures are made up of elements which are much smaller than the propagating EM wavelength (\(<<\lambda\)).

- MTM can exhibit simultaneously negative electric permittivity \(\varepsilon\) and magnetic permeability \(\mu\), and thus a negative refractive index (NRI).

- MTM refocuses incident radiation and can manipulate near field radiation.

MTM Phenomena
- Double Negative media (DNG)
- Negative Refractive Index (NRI)
- Left Handed Rule Wave Vector (LH)
- Backward Wave Phase Velocity (BW)
Let’s begin with a little history

How did we get started on the DNG stuff? What would they do for us once we have them?

Engineered media that have a negative index of refraction (negative permittivity and permeability)


Perfect reconstruction, high transverse wave vectors
Imaginary longitudinal component evanescent fields

The ‘Perfect Lens’
Imaging with DNG Lens

Field distribution along $z$ in the RHS of Lens

or ?
Ez Magnitude at 31.5 GHz
DOES IT WORK IN REAL-LIFE SCENARIO?

\[ \varphi \approx n k_0 d \]

\[ \varphi \text{ negative} \rightarrow n \text{ is negative} \rightarrow n_{eff} \text{ and } \varepsilon_{eff} \text{ are both negative} \]
MOTIVATION

Concept of the high gain antenna radome (1)

- Snell’s Law, $n_1 \sin \theta_1 = n_2 \sin \theta_2$
- By using of meta-materials, the effective refractive index could be less than unity
TYPICAL DNG CELL
DNG Superstrate

3D view

XZ cut

0.4 \lambda
By tuning the substrate thickness and metallic strip gap the smooth performance of reflective index could be obtained at 5.8 GHz band.
Constructed prototype(3)-simulated result

Max. gain of proposed antenna : 8.6 dBi@5.8 GHz
Max. gain of a patch : 5.1 dBi@5.8 GHz
Gain improvement : 3.5 dB
(Cal. By Ansoft HFSS Realized gain)
We find that the ray picture in an effective medium, used to predict the performance of the antenna/metamaterial composite, is not the correct one to use for this antenna configuration.
Geometry of a fabricated dipole strip FSS composite and its unit cell.
Directivity Comparison

![Graph showing directivity comparison between Dielectric, FSS, and DNG materials across different frequencies.](image_url)

(a) Dielectric Slab
(b) FSS
(c) DNG