Forum for Electromagnetic Research Methods and Application Technologies (FERMAT)

A tutorial on electromagnetic units and Constants

By

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Abstract: This discussion begins by presenting one way of looking at the nature of the scientific process, and explains how models and hence quantities are fundamental to this process. Since quantities and measurability go hand-in-hand, units, which are just references for quantities, become inevitable, as 'we can do no more than compare one thing with another.' Constants enter into the picture, as we need definitive numbers for our quantities, rather than just proportionalities, in the process of measurement.

After discussing the above, the presentation considers Maxwell's equations in generalized units, and explains how Maxwell deduced that light was electromagnetic in nature. It then goes on to explain how electrical units can be deduced in SI and CGS systems, and illustrates these with examples. A consideration of the nature of ε_o and μ_o follows. The presentation concludes with a thought on how a contextual consideration of electromagnetic theory can throw light on scientific approach.

Keywords: CGS units, Electromagnetic units, Maxwell's equations, Permittivity of free space, Permeability of free space, SI units, Units, Velocity of light

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KRISHNASAMY T. SELVAN obtained his BE (Hons), MS and PhD degrees respectively from Madurai Kamaraj University, Madurai (1987), Birla Institute of Technology and Science, Pilani (1996) and Jadavpur University, Kolkata (2002). He also obtained a PG Certificate in Higher Education from the University of Nottingham in 2007.

Selvan has been a Professor in the Department of Electronics and Communication Engineering, SSN College of Engineering, India, since June 2012. From early 2005 to mid-2012, he was with the Department of Electrical and Electronic Engineering, University of Nottingham Malaysia Campus. He also held the positions of the Assistant Director of Teaching and Learning for the Faculty of Engineering and the Deputy Director of Studies of the Department of Electrical and Electronic Engineering.

From early 1988 to early 2005, Selvan was with SAMEER – Centre for Electromagnetics, Chennai, India. Here he was essentially involved in antenna analysis, design, and testing. During 1994–1997, he was the Principal Investigator of a collaborative research programme that SAMEER had with the National Institute of Standards and Technology, USA. Later he was the Project Manager/Leader of some successfully completed antenna development projects. In early 1994, he held a two-month UNDP Fellowship at the RFI Industries, Australia.

Selvan's professional interests include electromagnetics, horn antennas, printed antennas, and electromagnetic education. In these areas, he has authored or coauthored a number of journal and conference papers. Selvan was on the editorial board of the International Journal of RF and Microwave Computer-Aided Engineering during 2006 to 2011. He was an academic editor for the International Journal on Antennas and Propagation from its inception in 2006 till 2014. He has been a reviewer for major journals including the IEEE Transactions on Antennas and Propagation. He was Technical Programme Committee co-chair for the IEEE Applied Electromagnetics Conference held in Kolkata in December 2011, and Student Paper Contest co-chair for IEEE AEMC 2013 held in Bhubaneswar. He was Publications Chair for the IEEE MTT-S International Microwave and RF Symposium (IMaRC) held in Bangalore in December 2014. He co-organized sessions on EM/microwave education during IMaRC 2014 and International Symposium on Antennas and Propagation, Kochi, 2014.

Selvan founded the Madras Chapter of the IEEE Antennas and Propagation Society (AP-S) in 2013, and has been the Chapter Chair since then. Selvan is a member of the Education Committee of the IEEE Antennas and Propagation Society. He is an AP-S Region 10 Distinguished Speaker for 2015-16.

Selvan is a senior member of the IEEE, a Fellow of the Higher Education Academy (UK), and a Life Member of the Society of EMC Engineers (India).

What is discussed?

- Why we have 2π's and 4π's in EM equations
- What ε_o and μ_o are, and what they are not
- Why is it that

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

- How Maxwell deduced that light is electromagnetic in nature
- The nature of scientific process

To that end we will discuss…

- Science
  - Quantities and measurements
    - Units and constants
      - Dimensions
- Rationalization
- Electromagnetic units
  - EM equations in generalized units
  - *c*, the velocity of light
  - SI and CGS units
- ‘Auxiliary’ field vectors
- Comments

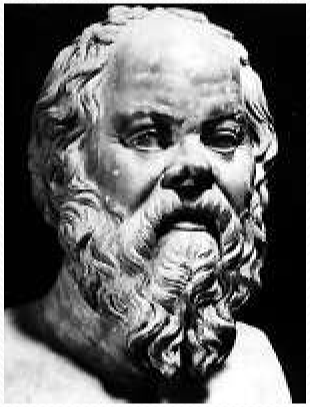
- Talk based on:

K.T. Selvan, "Fundamentals of electromagnetic units and constants," *IEEE Antennas and Propagation Magazine*, vol. 54, no. 3, pp. 100–114, June 2012

Science and measurements

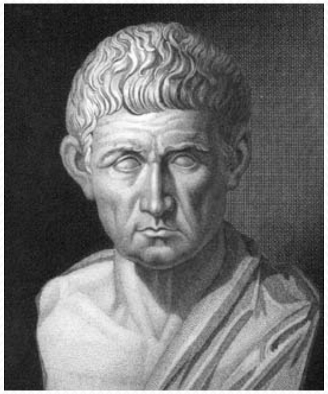
- This thing called science – a perspective:

Observation
⬇
Curiosity
⬇
Speculation
⬇
Quantity identification and Measurements
Quantities are definable and *measureable*
⬇
Modeling
Involves description of phenomena in terms of quantities
⬇
Refinements – may involve 'going back'
⬇
'Progress'



Socrates, 469 to 399 B.C.
http://www.philosophypages.com/ph/socr.htm

- Speculated that earth is spherical in nature



Aristotle, 384 to 322 B.C.
http://www.gap-system.org/~history/PictDisplay/Aristotle.html

- Provided a numerical estimate for the circumference of the earth

- Indian scripture *Vedanga Jyothisa*, believed to be dated between 1200 and 600 B.C., contained rules for *measuring* and fixing the days and hours

- Ancient Chinese took the value of the ratio of the diameter to the circumference of a circle as 3

Thus *quantity* is basic to science

- Quantities, along with their measurability, are fundamental to the pursuit of science

- Scientific enquiry needs
  - Identifying quantities
  - Equations relating quantities
    - Physical equations are *always* relations between measureable quantities

- ‘A **quantity …** is a quantifiable or assignable property ascribed to a particular phenomenon, body, or substance.’
  - Examples - mass of the moon, electric charge of the proton

http://physics.nist.gov/cuu/Units/introduction.html

- **Types**
  - Base - fully defined in physical terms, and *normally* regarded as independent of each other
    - Usually mass, length and time
  - Derived – Expressed in terms of base quantities
  - Practical and historical considerations dictate choice

So what are units?

- Measurement of quantities need references
  - 'We can do no more than compare one thing with another'

- We arbitrarily choose 'a particular sample of each kind of quantity' as the reference or the physical *unit* for that quantity.

And constants?

- Original experimental findings always presented as proportionalities rather than as equalities:
  - Coulomb: “The repulsive force…is in the inverse ratio of the square of the distances”
  - Faraday: “The chemical power of a current of electricity is in direct proportion to the absolute quantity of electricity which passes”
- We introduce constants to get equations
- Equations relating quantities also relate units and constants

Dimensions

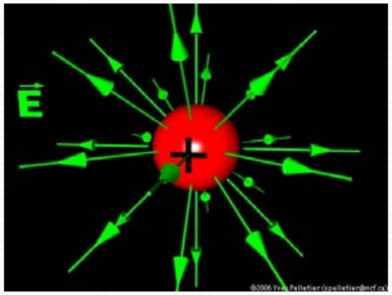
- Initiated by Fourier in 1822
- For a base quantity, a label of convenience
- For derived quantity, says how it is defined and how it changes when the size of the base units change:

$$\dim Q = L^{\alpha} M^{\beta} T^{\gamma}$$

  - α, β, and γ are small integers – positive, negative or zero
- A relative matter
  - But perspectives differ!
- Dimensional correctness of equations required

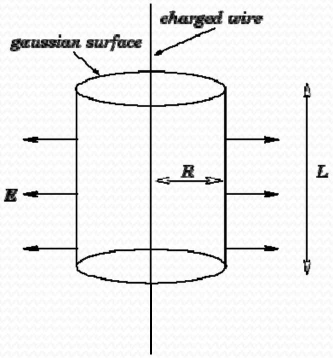
Rationalization

- Normal expectation:
  - '4π' in situations involving spherical symmetry



http://web.ncf.ca/ch865/englishdescr/EFldChargedSphere.html

  - '2π' in those involving cylindrical symmetry



http://farside.ph.utexas.edu/teaching/302l/lectures/node26.html

- Many classical formulas did not satisfy this expectation
  - In CGS units, for example, capacitance formulas:

$$C = \varepsilon_r \frac{A}{4\pi d}$$ Parallel-plate

$$C = \varepsilon_r \frac{ab}{b-a}$$ Spherical

- Heaviside
  - complained in 1882 about the 'eruption' of '4π's in EM equations
  - initiated the process of 'rationalizing' electromagnetic equations
- Process involves appropriate choice of constants

EM equations in generalized units

- We start with the fundamental units of mass (m), length (l) and time (t)
- Any secondary concept to be in the form of algebraic expression comprising the base quantities

- We start with Coulomb's electrostatic force law:

$$F_e \propto \frac{qq'}{r^2} \tag{1}$$

- More than 50 later, Faraday introduced a medium-dependent 'factor of ignorance,' k_1, as an empirical result:

$$F_e = k_1 \frac{qq'}{4\pi r^2} \tag{2}$$

- Ampere's law for force between two infinitely long parallel wires:

$$\frac{dF_m}{dl} = \frac{k_2}{2\pi}\frac{ii'}{d} \qquad (3)$$

- Charge-current relationship:

$$i = k_3 \frac{dq}{dt} \qquad (4)$$

- Force laws (2) and (4) need to be dimensionally equivalent:

$$[k_1]\frac{[q]^2}{L^2} = [k_2][k_3]^2 \frac{[q]^2}{T^2}\frac{L}{L} \qquad (5)$$

- We obtain from (5)

$$\frac{[k_1]}{[k_2][k_3]^2} = \frac{L^2}{T^2} = [v]^2 \qquad (6)$$

- (6) is the ratio of electrostatic to electromagnetic forces, and is a velocity

- **Definition** of field quantities:

$$E = k_1 \frac{q}{r^2} \qquad B = \frac{k_2}{2\pi}\frac{i}{d} \qquad (7)$$

- Maxwell's equations in *generalized* units:

Gauss's law

$$\nabla \cdot \mathbf{E} = k_1 \rho \qquad (8)$$

Ampere-Maxwell law

$$\nabla \times \mathbf{B} = k_2 \mathbf{J} + \frac{k_2 k_3}{k_1} \frac{\partial \mathbf{E}}{\partial t} \qquad (9)$$

Law of no magnetic monopoles

$$\nabla \cdot \mathbf{B} = 0 \qquad (10)$$

Faraday's law

$$\nabla \times \mathbf{E} = -k_3 \frac{\partial \mathbf{B}}{\partial t} \qquad (11)$$

- Maxwell's equations can be combined to form the electromagnetic wave equation:

$$\nabla^2 W = \frac{k_3^2 k_2}{k_1} \frac{\partial^2 W}{\partial t^2} \qquad (12)$$

- Standard equation of wave motion is:

$$\nabla^2 W = \left(\frac{1}{v^2} \right) \frac{\partial^2 W}{\partial t^2} \qquad (13)$$

- Comparing (12) with (13), the EM wave travels with a velocity

$$v = \frac{1}{k_3} \sqrt{\frac{k_1}{k_2}} \qquad (14)$$

  - (14) is the same as (6), the ratio of EM forces!

Velocity of EM wave

- Weber's force law also contains the ratio of electrostatic and electromagnetic forces (c_w):

$$F = \frac{qq'}{r^2}\left(1 - \frac{v^2}{c_w^2} + \frac{2ra}{c_w^2}\right)$$

- Weber and Kohlrausch made the first numerical determination of this velocity (ratio), obtaining a value of 3.1074×10^8 m/s
- Foucault later measured the velocity of light and obtained a value of 2.98×10^8 m/s

- Maxwell deduced that light is an electromagnetic disturbance, noting
  - the close agreement of the above values with each other
  - that Weber and Kohlrausch did not use light except to see instruments
  - that Foucault's method made no use of electricity and magnetism
- We can rewrite (14) as

$$c = \frac{1}{k_3}\sqrt{\frac{k_1}{k_2}} \qquad (15)$$

- The modern value for c is $c = 2.997930 \times 10^8$ m/s.

We are now back: Electromagnetic units

- Constants: c, k_1, k_2, k_3

- c is known. Two of the other three can be chosen arbitrarily, with the third one determined through (15)

- Numerous choices possible

Constant

New quantity

$$F_e = k_1 \frac{qq'}{4\pi r^2}$$

Definable in terms of base quantities

Base quantity

Coulomb’s force law

- Electromagnetic unit systems in common use:
  - Gaussian (CGS) units
    - $k_1 = 4\pi$; q is expressible purely in terms of mechanical quantities
  - International System of Units (SI)
    - Introduces a fourth fundamental unit of electrical nature
    - Thus electrical and mechanical forces are distinguished

- Choice of constants dictates the unit system and hence…
  - does not have fundamental significance!

- CGS system is useful in microscopic problems involving the electrodynamics of individual charged particles

- SI system is useful in practical, large-scale phenomena, especially in engineering applications

SI system

- Proposal made by Giorgi in 1901
- Adapted by the International Electrotechnical Commission (IEC) in 1935
- Fundamental quantities: mass (kg), length (m), time (s), current (A)
- Ampere definition:
    - "One ampere is that constant electric current which, if maintained in two straight parallel conductors of infinite length, of negligible cross-section, and placed one meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length."
  - Thus actually a derived quantity!

- Reconsider Ampere's force law (3):

$$F_m = \frac{k_2}{2\pi}\frac{I^2}{r}$$

- Employment of Ampere's definition leads to:

$$k_2 = 4\pi \times 10^{-7} \quad \text{N/A}^2$$

- N/A^2 in base units is $\text{kg}\cdot\text{m}\cdot\text{s}^{-2}\cdot\text{A}^{-2}$; equivalently, H/m
- $k3$ chosen to be of unit magnitude and dimensions
- Readily, $k_1 = 4\pi\times10^{-7}c^2$, with the unit of $\text{kg}\cdot\text{m}^3\cdot\text{s}^{-4}\cdot\text{A}^{-2}$
- Units for derived quantities can now be obtained

- **Illustrations:**

  - Electric charge

$$F_e = k_1 \frac{qq'}{4\pi r^2}$$

$$[q]^2 = \frac{[F_e]L^2}{[k_1]} = \frac{MLL^2}{T^2 M L^3 T^{-4} I^{-2}}$$

$$[q] = IT$$

  - The corresponding unit is s.A, having the special name of Coulomb (C)

- Voltage

$$V = Er = k_1 \frac{q}{r}$$

$$[V] = [k_1]\frac{[q]}{[r]} = L^3MT^{-4}I^{-2}\frac{IT}{L} = L^2MT^{-3}I^{-1}$$

- The corresponding unit is $m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$, which has been assigned the special symbol V

- Magnetic flux density

$$B = \frac{k_2}{2\pi}\frac{i}{d}$$

$$[B] = \frac{[k_2]I}{L} = \frac{MLT^{-2}I^{-2}I}{L} = \frac{M}{IT^2}$$

- The corresponding unit is $kg \cdot s^{-2} \cdot A^{-1}$, or Wb/m^2

Gaussian (CGS) system

- $c = 2.997930 \times 10^{10}$ cm/s
- $k_1 = k_2 = 4\pi$
- Then, $k_3 = 1/c$

- Electric charge

$$F_e = k_1 \frac{qq'}{4\pi r^2} = \frac{qq'}{r^2}$$

$$[q]^2 = [F_e]L^2 = ML^3T^{-2}$$

- The corresponding CGS units of electric charge is $g^{1/2}\cdot cm^{3/2}\cdot s^{-1}$. This unit is called statcoulomb.

‘Auxiliary’ field vectors and EM constants

Free space electromagnetics
E, B

Fields in material medium
D, H

- Linear, isotropic media:
  - Polarization P – dielectrics
  - Magnetization M – magnetic materials

$$\mathbf{D} = \frac{1}{k_1}\mathbf{E} + \mathbf{P} \rightarrow \frac{1}{k_1}\mathbf{E} + \alpha\mathbf{E} \rightarrow \mathbf{E}\frac{1}{k_1}(1 + k_1\alpha) \rightarrow \varepsilon\mathbf{E}$$

$$\mathbf{H} = \frac{1}{k_2}\mathbf{B} - \mathbf{M} \rightarrow \frac{1}{k_2}\mathbf{B} - \beta\mathbf{H}$$

$$\Rightarrow \mathbf{B} = k_2\mathbf{H}(1 + \beta) \rightarrow \mu\mathbf{H}$$

$$\boxed{\varepsilon = \frac{1}{k_1}(1 + k_1\alpha)} \qquad \boxed{\mu = k_2(1 + \beta)}$$

Material medium

$$\varepsilon = \frac{1}{k_1}(1 + k_1\alpha)$$

Material medium

$$\mu = k_2(1 + \beta)$$

$$\varepsilon_o = \frac{1}{k_1}$$

Free-space value

$$\mu_o = k_2$$

Free-space value

- ε_o and μ_o thus constants of proportionality
  - They do not represent any property of free space
- '**Electric constant**' for ε_o and '**magnetic constant**' for μ_o have been proposed
- Characteristic impedance of free space

$$\eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}}$$

is a more tangible physical quantity

- Perspectives on the nature of field vectors:
  - **E**, **P**, **B**, and **M** are only fundamental
  - All four vectors are basic
- Differences in perspectives
  - Inevitable
  - Desirable

Some comments

- Keeping an eye on fundamentals
- Levels of intellectual development [M.B. Magolda, 1992]:
  - Absolute knowing: certainty of all knowledge
  - Transitional knowing: some knowledge certain, some not
  - Independent knowing: most knowledge uncertain
  - Contextual knowing:
    - all knowledge is contextual and individually constructed
    - Open to changing conclusions in the face of new evidence

- Innovation often demands adaptability, rather than rigidity, of ideas, to evolving research paradigms
- Science does not develop in simplistic way
- Variation in perspectives are widely (and inevitably) prevalent and are desirable in research
- The image of the certainty of scientific knowledge is not to be taken for granted

Thank you!