Forum for Electromagnetic Research Methods and Application Technologies (FERMAT)

A Story of Maxwell's Displacement Current

By

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Abstract: This presentation discusses the story of Maxwell's displacement current, acknowledged as one of the greatest innovations introduced in the physical sciences. Starting with a discussion on the text-book version of how the term was introduced, and on the significance of the term, the presentation proceeds to give a historical account on how, and in what context, the term was actually introduced by Maxwell. The basic questions on the topic that have inspired, and continue to inspire, several papers are then stated, and ways of looking at their answers considered. In the process, the presentation attempts to highlight how the displacement current concept, considered in context, can throw light on elements of creativity, life-long learning skill, and scientific approach, aspects that are much talked about these days. Some student comments obtained after this presentation had been made to the presenter's class are also included.

Keywords: Ampere's law, Coulomb gauge, Displacement current, EM teaching, Lorentz gauge, Maxwell's equations, Molecular-vortex model

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From early 1988 to early 2005, Selvan was with SAMEER – Centre for Electromagnetics, Chennai, India. Here he was essentially involved in antenna analysis, design, and testing. During 1994–1997, he was the Principal Investigator of a collaborative research programme that SAMEER had with the National Institute of Standards and Technology, USA. Later he was the Project Manager/Leader of some successfully completed antenna development projects. In early 1994, he held a two-month UNDP Fellowship at the RFI Industries, Australia.

Selvan's professional interests include electromagnetics, horn antennas, printed antennas, and electromagnetic education. In these areas, he has authored or coauthored a number of journal and conference papers. Selvan was on the editorial board of the International Journal of RF and Microwave Computer-Aided Engineering during 2006 to 2011. He was an academic editor for the International Journal on Antennas and Propagation from its inception in 2006 till 2014. He has been a reviewer for major journals including the IEEE Transactions on Antennas and Propagation. He was Technical Programme Committee co-chair for the IEEE Applied Electromagnetics Conference held in Kolkata in December 2011, and Student Paper Contest co-chair for IEEE AEMC 2013 held in Bhubaneswar. He was Publications Chair for the IEEE MTT-S International Microwave and RF Symposium (IMaRC) held in Bangalore in

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Selvan is a senior member of the IEEE, a Fellow of the Higher Education Academy (UK), and a Life Member of the Society of EMC Engineers (India).

Agenda

- The displacement current and its significance
- Evolution of the concept
- Fundamental questions. Answers?
- Appreciation and depreciation
- To reflect on
  - Elements of creativity
  - Qualities necessary for nourishing it
- How EM theory discussed in context could help with modern educational objectives:
  - Openness, life-long learning, creativity

- **Talk based on:**

K.T. Selvan, “A revisiting of scientific and philosophical perspectives of Maxwell’s displacement current,” *IEEE Antennas and Propagation Magazine*, vol. 51, no. 3, pp. 36–46, June 2009

EM laws till 1861

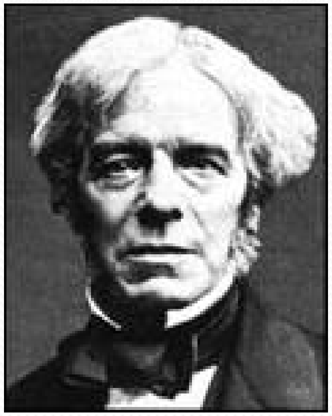
Coulomb

Gauss

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

Faraday



$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_C$$

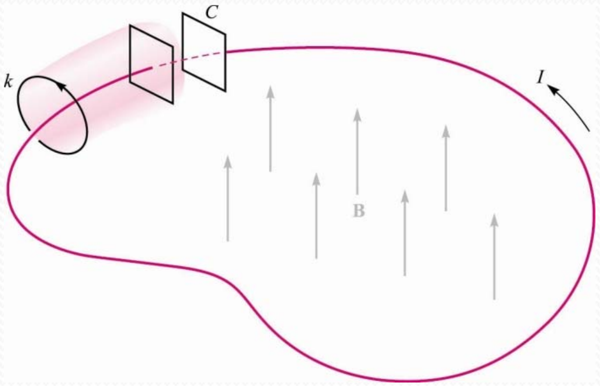
Ampere

Limitations of Ampere’s law

Maxwell recognized that Ampere’s law

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I$$

does not hold for open circuits:



A conducting loop connecting the two plates of a capacitor

- Also consider this:

$$\nabla \times \mathbf{H} = \mathbf{J}$$

Take divergence on either side:

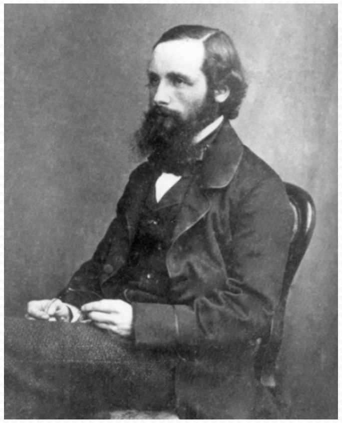
$$\nabla \cdot \nabla \times \mathbf{H} \equiv 0 = \nabla \cdot \mathbf{J}$$

But

Equation of continuity $$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

Implication?

How Maxwell fixed Ampere’s law – Textbook version



- Add an unknown term **G** to Ampere’s law:

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{G}$$

- Take divergence:

$$0 = \nabla \cdot \mathbf{J} + \nabla \cdot \mathbf{G}$$

- Therefore:

$$\nabla \cdot \mathbf{G} = \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t}(\nabla \cdot \mathbf{D}) = \nabla \cdot \frac{\partial \mathbf{D}}{\partial t}$$

- Thus:

$$\mathbf{G} = \frac{\partial \mathbf{D}}{\partial t}$$

With this addition, Ampere's law now becomes Ampere-Maxwell law:

Ampere-Maxwell law $$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (4)$$

$\partial \boldsymbol{D}/\partial t$ is displacement current density

- This equation is consistent with the continuity equation
- It also satisfies open circuit conditions

Maxwell’s equations consolidated

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_C + \boxed{\varepsilon \frac{\partial \mathbf{E}}{\partial t}}$$

Displacement current density

$\partial \mathbf{D}/\partial t$: Significance and implications

- The introduction of the term:
  - Brought about a symmetry in Maxwell's equations:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \mathbf{J}_C + \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

- It led to a path-breaking prediction:
  - Electromagnetic wave can be generated and propagated
- Other implications of Ampere-Maxwell equation:
  - The displacement current causes a magnetic field
  - It is equivalent to a *current*

How Maxwell *actually* fixed Ampere’s law?

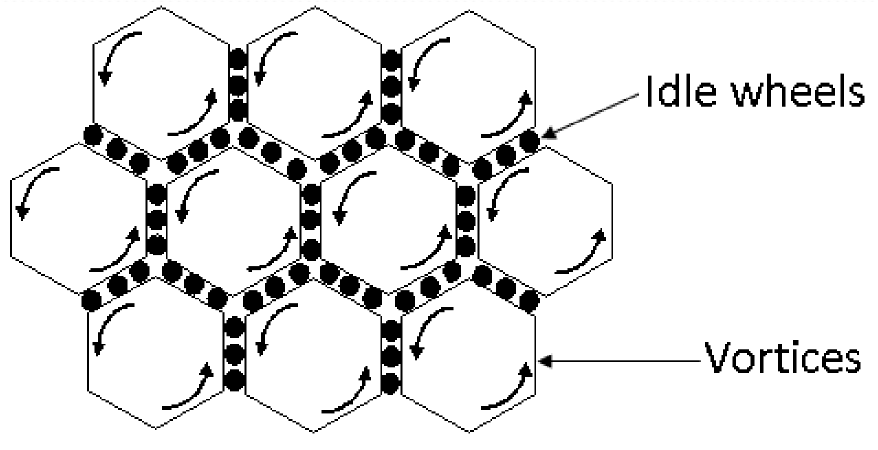
The context: A medium

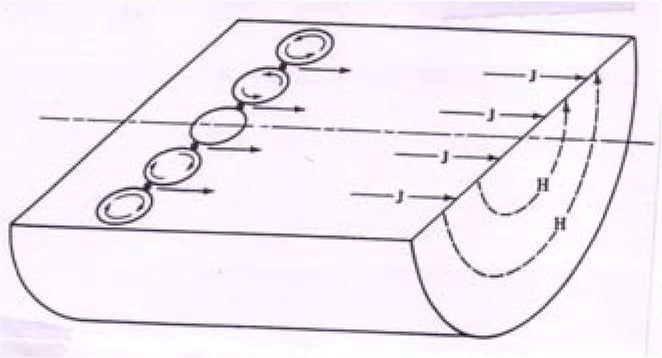
- 17th century philosophers:
    - understanding of nature calls for mechanical explanations
- Maxwell shared this belief
- He therefore rejected Weber's action-at-a-distance theory, *admitting* though that the two approaches were mathematically equivalent
- He believed that
    - 'it is a good thing to have two ways of looking at a subject, and to admit that there are two ways of looking at it'

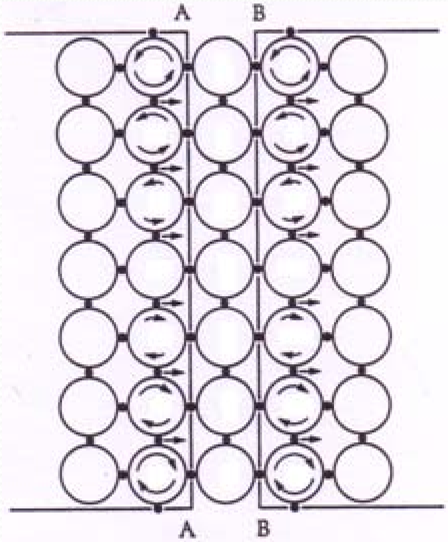
$\partial \mathbf{D}/\partial t$: Motivation

- Introduced in 1862
- Original motivation [Siegel, 1991]:
  - To extend the molecular –vortex theory to account for charge accumulation at conductor-dielectric boundary
  - To ensure consistency of Ampere's law with the continuity equation
  - To generalize Ampere's law to be applicable to open circuits
- Reformulated in 1873 so as to be independent of the molecular-vortex theory

The molecular vortex model







D.M. Siegel, *Innovation in Maxwell's Electromagnetic Theory*

How the concept evolved

- Mechanical framework:
  - Belief in analogy first and conviction in mechanical representation then
- Introduced in 1862:
  - "Electromotive force acting on a dielectric produces a state of polarization of its parts ....The effect of this action on the whole dielectric mass is to produce a general displacement of the electricity in a certain direction"

- Maxwell’s justification:
- “… the extreme difficulty of reconciling the laws of electromagnetism with the existence of electric currents which are not closed is one reason among many why we must admit the existence of transient currents due to the variation of displacement.”

- What he *believed*:
  - "electric displacement. ..is a movement of electricity in the *same sense* as" the normal "movement of electricity"

Maxwell's changing ideas

- Though committed to mechanical approach in his 'period of intensive innovation,' Maxwell displayed changing attitude
- Initial belief: Physical analogies
    - 'Faraday's lines of force,' 1855
- Middle period: Need for medium
    - Initial formulation of displacement current
- Later period: Skeptical towards mechanical representation
    - 'Treatise,' 1873

Philosophical highlights

- The discovery of displacement current represents a major, theoretically motivated innovation
- A brilliant electrical discovery arrived at based on mechanical considerations
- A very good illustration of the way of innovators
- Much admired
- Also criticized!

The debate

Fundamental questions

- Does the displacement current produce a magnetic field?
- Is the displacement current equivalent to an electric current?

Coulomb and Lorentz gauges

- Mathematical procedure
- Coulomb gauge:
  - Given by $\nabla \cdot \mathbf{A} = 0$
  - Predicts instantaneous potentials
  - Predicts fields that propagate with finite velocity
- Lorentz gauge:
  - Given by

$$\nabla \cdot \mathbf{A} = \mu\varepsilon \frac{\partial \phi}{\partial t}$$

  - Predicts retarded potentials
  - Predicts fields that propagate with finite velocity

Does $\mathbf{J}_D$ produce magnetic field?

- Purcell:

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t} = \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{J}_d = \nabla \times \varepsilon \frac{\partial \mathbf{E}}{\partial t} = \varepsilon \frac{\partial}{\partial t}(\nabla \times \mathbf{E}) = -\varepsilon \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

- For slowly varying fields, $\nabla \times \mathbf{J}_d = 0$. Hence $\nabla \cdot \mathbf{J}_d$ should be non-zero
- Magnetic field of any radial, symmetrical current distribution must be zero by symmetry

- Rosser:
  - Using certain substitutions and the Coulomb gauge, the displacement current can be eliminated from the wave equation for **A**:

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu_o\mathbf{J}_C + \mu_o\varepsilon_o\nabla\frac{\partial\phi}{\partial t}$$

$$= -\mu_o\left(\mathbf{J}_\mathbf{C} - \varepsilon_o\nabla\frac{\partial\phi}{\partial t}\right) = -\mu_o\mathbf{J}_t$$

  - $\mathbf{J}_t$ is transverse current density, calculable using conduction current and scalar potential
  - Thus no part of the displacement current gives rise to a magnetic field!

- Zapolsky:

  - Rosser’s argument is semantic. By appropriate substitutions, the wave equation can be written as

$$\nabla^2 \mathbf{A} = -\mu_o \left(\mathbf{J}_C + \frac{\partial \mathbf{D}}{\partial t} \right)$$

  - The displacement current is thus hidden in Rosser’s equation!

- Jackson:
  - Capacitor magnetic field estimation by using Ampere’s law requires
    - the inclusion of displacement current to obtain correct result for arbitrary choices of contours
    - only the conduction currents for certain contours
- Roche:
  - The displacement current of a rapidly changing induced electric field generates a significant magnetic field

- Hertz’ experimental demonstration

Is $\mathbf{J}_D$ equivalent to current?

- Roche:
  - In the Coulomb gauge, the displacement current can be considered to be equivalent to an electric current
  - But the Coulomb gauge is not a physical gauge and the equivalence of $\partial\mathbf{D}/\partial t$ to current is always purely fictional
  - Term very important, but the name should **not** be mentioned to students, as it is in no sense an electric current!

- Jackson:
  - The Coulomb gauge is no more or less physical than any other
  - Maxwell is wrong (only) if he asserts that the displacement current is a real external current density on a par with the conduction current density, but he is right if he says that it is electromagnetically equivalent to current
- Yaghjian:
  - There is mathematically nothing wrong with the Coulomb gauge

Re-asking the questions…

- Does the displacement current produce a magnetic field?
- Is the displacement current equivalent to an electric current?
  - There is only one answer to both!

Deductions and summing up

On the concept

- The displacement current concept is ‘closest to…a genuine, useful, profound theory…built purely *speculatively*’ [Einstein]
- Without the displacement current, the treatment of electromagnetic waves would have been absurdly complicated [French and Tessman]
- Its development surrounded by a variety of positive and negative discussion
- But then that is the way of science and human thought – and hence of innovation!

What we learn

- Innovation often demands adaptability, rather than rigidity, of ideas, to evolving research paradigms
- Science does not develop in simplistic way
- Variation in perspectives are widely (and inevitably) prevalent and are desirable in research
- The image of the certainty of scientific knowledge is not to be taken for granted

Pedagogical implications

- Levels of intellectual development [M.B. Magolda, 1992]:
  - Absolute knowing: certainty of all knowledge
  - Transitional knowing: some knowledge certain, some not
  - Independent knowing: most knowledge uncertain
  - Contextual knowing:
    - all knowledge is contextual and individually constructed
    - Open to changing conclusions in the face of new evidence
- Maxwell's theory, his approach and thoughts present a fertile ground for accommodating the highest level

Some student feedback

- ‘The term might be confusing at first. But through further discussion, it enhances understanding of Maxwell’s equations.’
- ‘Displacement current somewhat helped my understanding in electromagnetism.’
- ‘The historical aspects covered in the explanation of displacement current theory motivates myself to have an open minded approach to education.’
- ‘Introducing this concept was interesting and indeed a bit confusing. At least we can now do some research on the matter to gain a better understanding.’

Reflecting…

- It pays to discuss the term displacement current in context
- The story helps us contemplate about
  - Being open and receptive
  - Being reflective
  - Being a life-long learner!
- These constitute higher-order thinking skills