Introduction – Quantum Optics and Entanglement

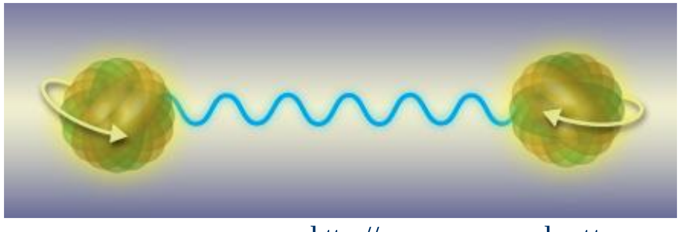
- Quantum optics refers to the study of non-classical light arising from quantized Maxwell's equations (single and few photons, vacuum fluctuations, spontaneous emission, etc.).

- Results in a fully quantum-dynamical model for both matter (e.g., electrons) and radiation (photons), which is necessary to study quantum entanglement.

Introduction – Quantum Entanglement

- Entanglement is an experimentally verified property of nature where pairs of quantum systems are "connected" in some manner such that the quantum state of each system cannot be described independently.

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\rightarrow\rangle_2 - |\rightarrow\rangle_1|\uparrow\rangle_2)$$



http://www.research.att.com

- Measurements on one system of a pair of entangled systems collapses the wavefuction of the entangled system, so that the other system appears to "know" what measurement was performed on the first system, instantaneously.

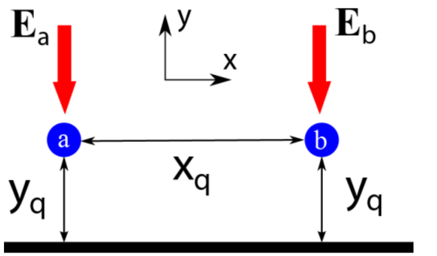
Introduction – Quantum Entanglement

However, this does not allow faster-than-light communications (i.e, measurer #1 can't control what is measured, resulting in the no-communication theorem and no-cloning theorem).

So, what is entanglement good for?

- Entanglement is the cornerstone of much of quantum computation and quantum information theory.
  - Generating, preserving, and controlling entanglement is necessary for many quantum computer implementations.

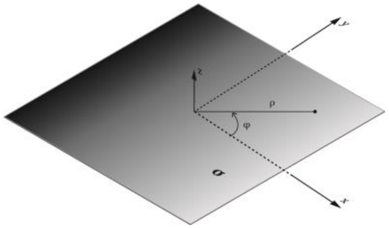
Motivation



It is highly desirable to electronically manipulate the photonic spectrum of a multi-level emitter such as an atom or quantum dot (QD), and to control entanglement, via a macroscopic, easily-adjusted external parameter (e.g., bias).

Surface plasmon polaritons (SPPs) on graphene are highly tunable, and offer a promising way to achieve electronic control over a quantum emitter mediated by graphene SPPs.

Graphene – Electromagnetic Modeling

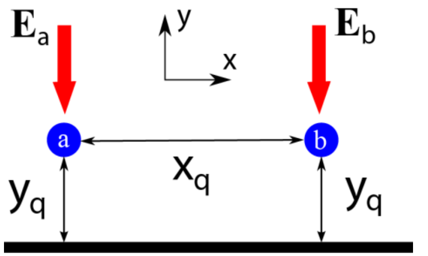


Infinite contiguous graphene sheet modeled as a two-sided impedance surface having conductivity σ (S).

$$\sigma(\omega) = \frac{ie^2 k_B T}{\pi\hbar^2(\omega + i\Gamma_1)}\left(\frac{\mu_c}{k_B T} + 2\ln\left(e^{-\frac{\mu_c}{k_B T}} + 1\right)\right)$$

$$+ \frac{ie^2(\omega + i\Gamma_2)}{\pi\hbar^2}\int_0^\infty \frac{f_d(-\varepsilon) - f_d(\varepsilon)}{(\omega + i\Gamma_2)^2 - 4(\varepsilon/\hbar)^2}d\varepsilon$$

Formulation



The Hamiltonian of the coupled system is the sum of QDs, pump, reservoir (graphene+vacuum), and their interaction

$$H = \int d\mathbf{r} \int_0^{+\infty} d\omega_\lambda \hbar\omega_\lambda \mathbf{b}^\dagger(\mathbf{r},\omega_\lambda,t)\cdot\mathbf{b}(\mathbf{r},\omega_\lambda,t) + \sum_{m=a,b} \hbar\omega_m \sigma_m^+(t)\sigma_m^-(t)$$

$$-\sum_{m=a,b} (\sigma_m^+(t)+\sigma_m^-(t))\mathbf{d}\cdot\mathbf{E}(\mathbf{r}_m,t)$$

$$\mathbf{E}(\mathbf{r},t) = i\int_0^\infty d\omega_\lambda \int \sqrt{\frac{\mathrm{Im}(\varepsilon(\mathbf{r}';\omega_\lambda))\hbar}{\pi\varepsilon_0}}\,\mathbf{G}(\mathbf{r},\mathbf{r}';\omega_\lambda)\cdot\mathbf{b}(\mathbf{r}',\omega_\lambda,t)d\mathbf{r}' + \mathrm{H.C.}$$

$$\nabla\times\nabla\times\mathbf{G}(\mathbf{r},\mathbf{r}') - k_0^2\varepsilon(\mathbf{r})\mathbf{G}(\mathbf{r},\mathbf{r}') = k_0^2\mathbf{I}\delta(\mathbf{r}-\mathbf{r}')$$ **Classical Green function**

σ^+ and σ^- are creation and annihilation operators for the atoms, **b** are creation and annihilation (bosonic) operators for the photons.

Formulation – Density Operator and Quantum Master Equation

Evolution of the system density matrix $\rho_s = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ is described by the Von Neumann equation, $\partial_t \rho_s = -(i/\hbar)[H, \rho_s]$

$$\partial_t \rho_s(t) = -(i/\hbar)[\hat{V}, \rho_s(t)] + \mathcal{L}\rho_s(t),$$

Evolution equation

$$\hat{V} = -\hbar \sum_{j=a,b} (\Omega_j e^{-i\Delta_j t} \hat{\sigma}_j^\dagger + \Omega_j^* e^{i\Delta_j t} \hat{\sigma}_j)$$

Source term

$$\mathcal{L}\rho_s = \sum_{i,j=a,b} \frac{\Gamma_{ij}(\omega_d)}{2} (2\hat{\sigma}_i \rho_s \hat{\sigma}_j^\dagger - \hat{\sigma}_i^\dagger \hat{\sigma}_j \rho_s - \rho_s \hat{\sigma}_i^\dagger \hat{\sigma}_j)$$
$$+ i[(g_{ab}(\omega_d)\hat{\sigma}_a^\dagger \hat{\sigma}_b + g_{ba}(\omega_d)\hat{\sigma}_b^\dagger \hat{\sigma}_a), \tilde{\rho}_s(t)],$$

$\mathcal{L}$, Lindblad superoperator

$\Omega_j = \mathbf{d} \cdot \mathbf{E}_j / \hbar$ (the effective Rabi frequency of the pump)

Formulation – Density Operator and Quantum Master Equation

$$\Gamma_{ij}(\omega_d) = \frac{2\omega_d^2}{\varepsilon_0 \hbar c^2} \operatorname{Im} \mathbf{d} \cdot \underline{\underline{\mathbf{G}}}(\mathbf{r}_i, \mathbf{r}_j, \omega_d) \cdot \mathbf{d},$$

$$g_{ij}(\omega_d) = \Delta_{ij} = \frac{\omega_d^2}{\varepsilon_0 \hbar c^2} \operatorname{Re} \mathbf{d} \cdot \underline{\underline{\mathbf{G}}}(\mathbf{r}_i, \mathbf{r}_j, \omega_d) \cdot \mathbf{d},$$

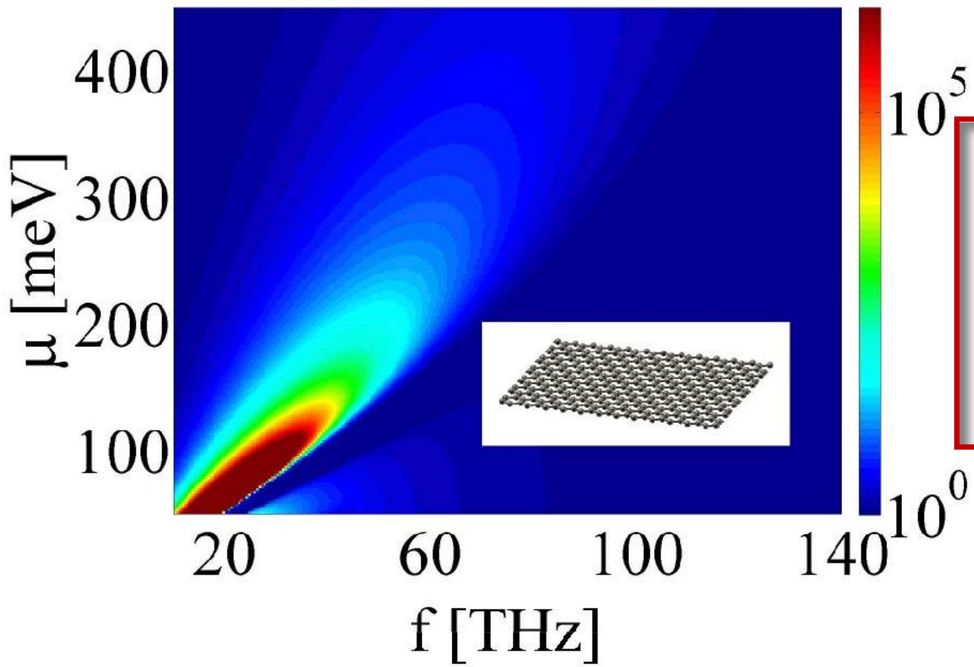
Γ_{ii} is the rate of spontaneous emission, related to the LDOS

Γ_{ij} ($i \neq j$) is a dissipative coupling term

g_{ij} is a coherent dipole-dipole coupling parameter

Formulation – Computation of Green Functions

- In general, we use the commercial FDTD code Lumerical to numerically compute the Green function.
- Allows for a true dipole source.
    - Avoids numerical issues involved with discretizing a small line of current (e.g., CST).
    - Works very well for a variety of nanostructures (plasmonic rods, grooves, arrays of nano-spheres, optical Yagi-Uda antennas, etc.)
    - Finite-sized graphene problematic, so we assume infinite graphene.



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Graphene as a tunable THz reservoir for shaping the Mollow triplet of an artificial atom via plasmonic effects

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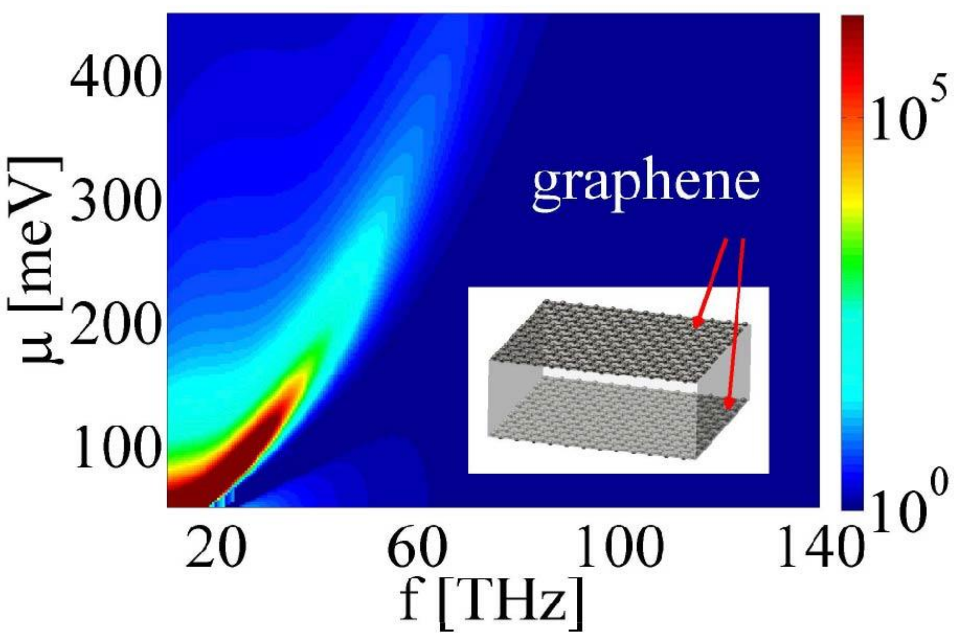
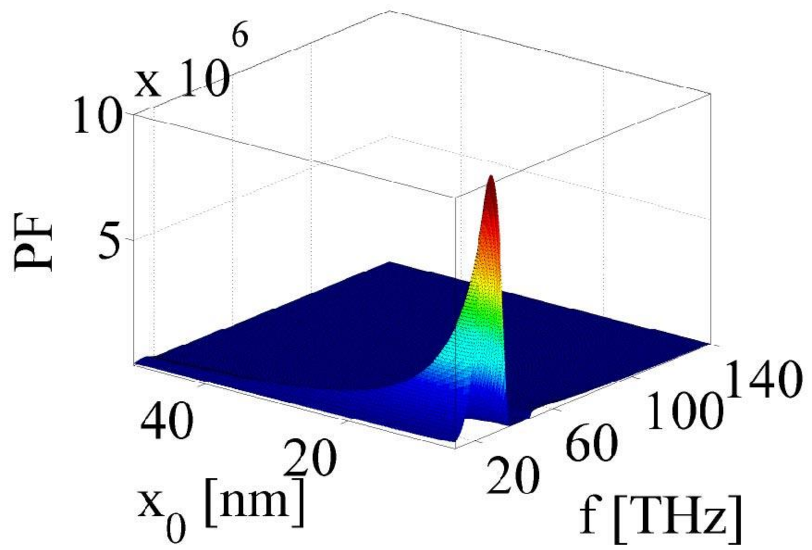
Purcell factor (PF), Γ_{ii}/Γ_0





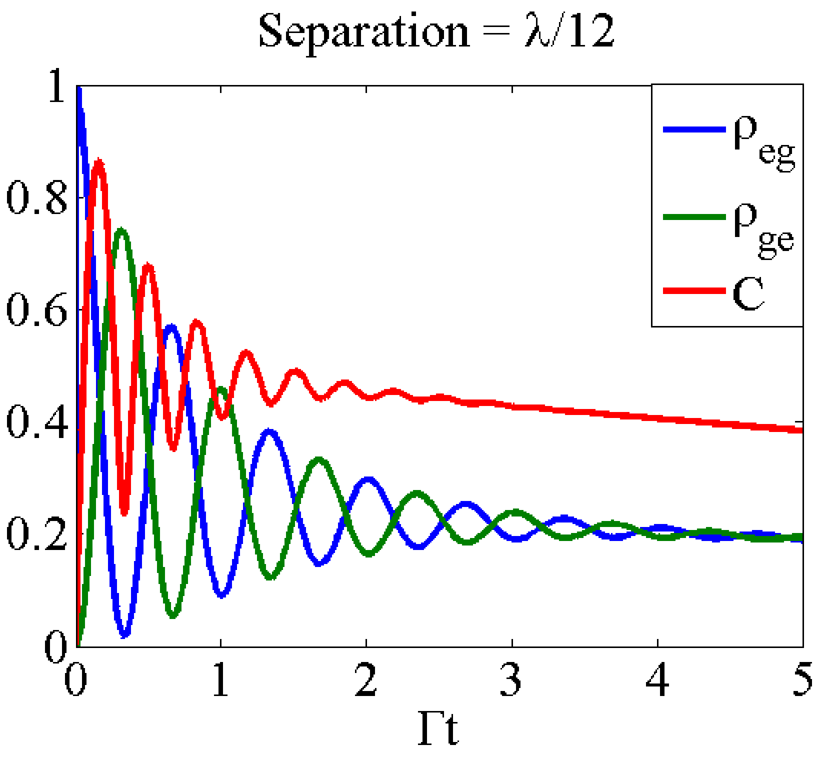
Entanglement

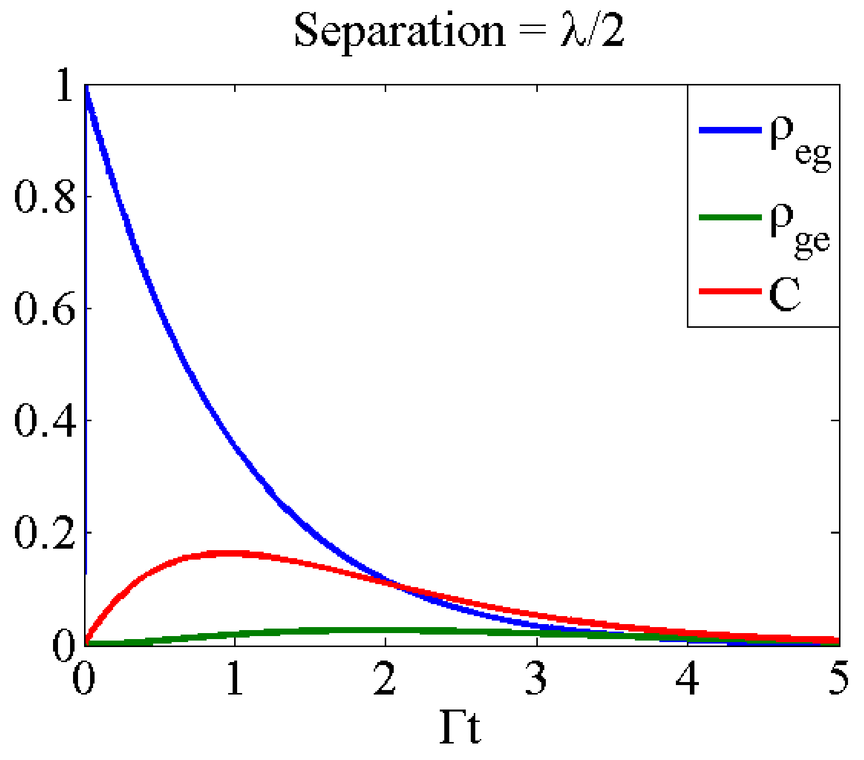
To assess entanglement we use the concurrence.
C=0: no entanglement,
C=1: maximum entanglement

$$C = \max(0, \sqrt{u_1} - \sqrt{u_2} - \sqrt{u_3} - \sqrt{u_4})$$

where u_i are arranged in the descending order of the eigenvalues of the matrix $\rho_s \tilde{\rho}_s$,
where $\tilde{\rho}_s = \sigma_y \otimes \sigma_y \hat{\rho}_s^* \sigma_y \otimes \sigma_y$, σ_y is the Pauli matrix.

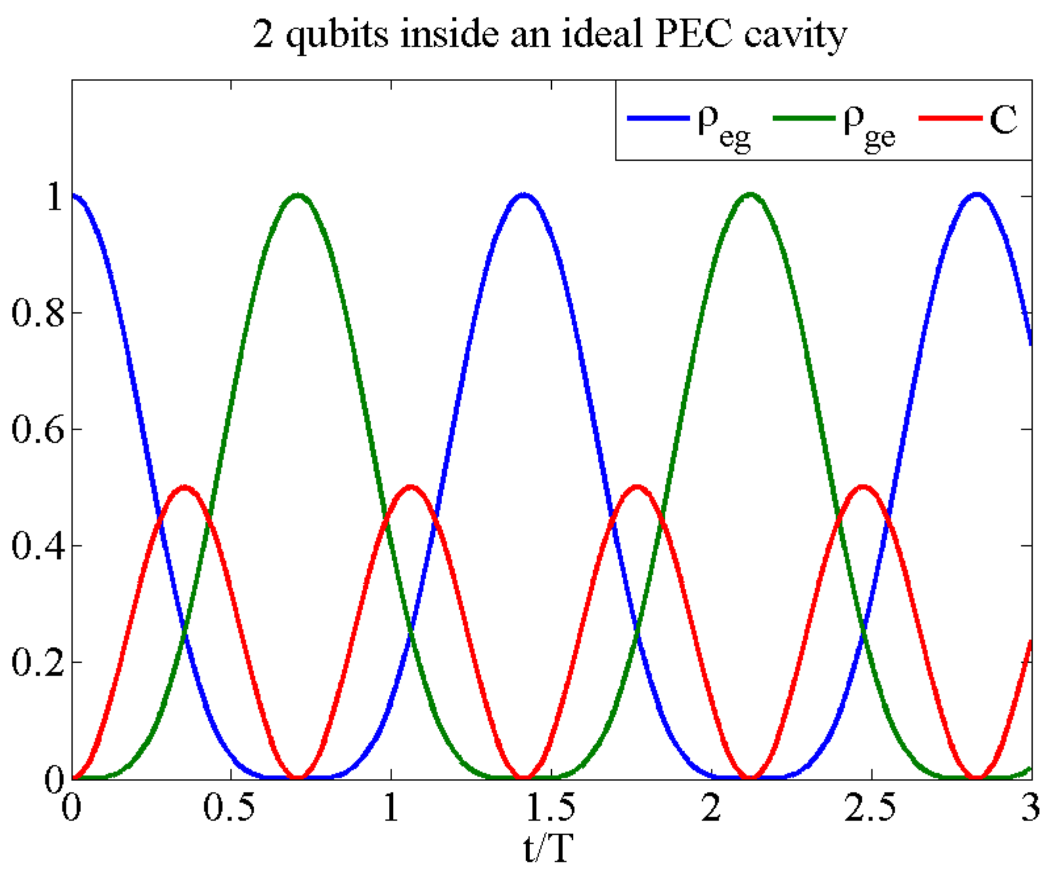
Transient Entanglement via Spontaneous Emission in Vacuum





Population dynamics and transient entanglement between two quantum emitters in vacuum.

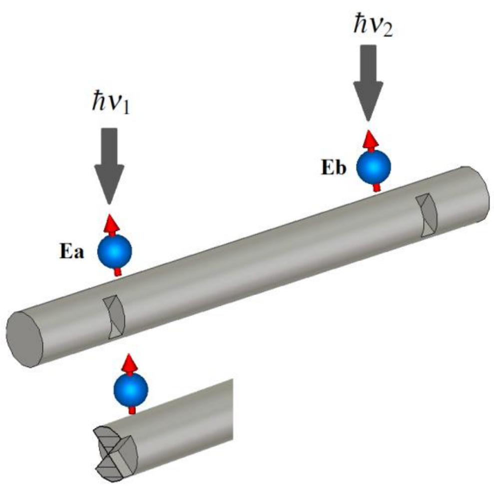
Transient Entanglement via Spontaneous Emission in PEC Cavity

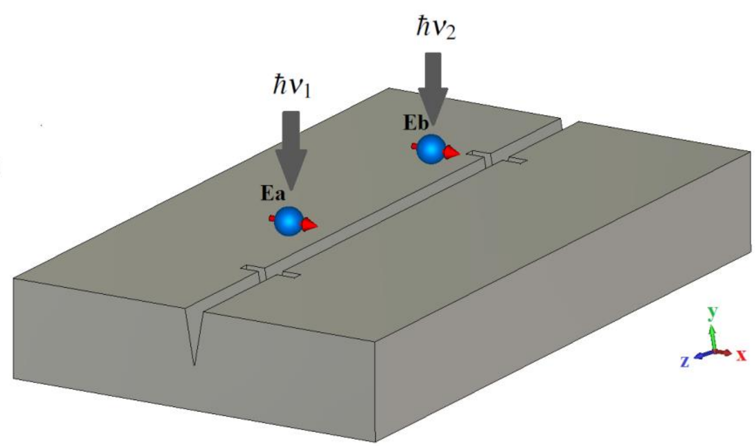


Population dynamics and transient entanglement between two quantum emitters in a PEC cavity, separation between emitters is λ/12 at 80 THz.

Graphene-Mediated Entanglement

- Entanglement is strong and relatively long-lived between closely-spaced emitters in vacuum.
- Plasmonic and other waveguiding systems can aid entanglement between far-separated emitters.

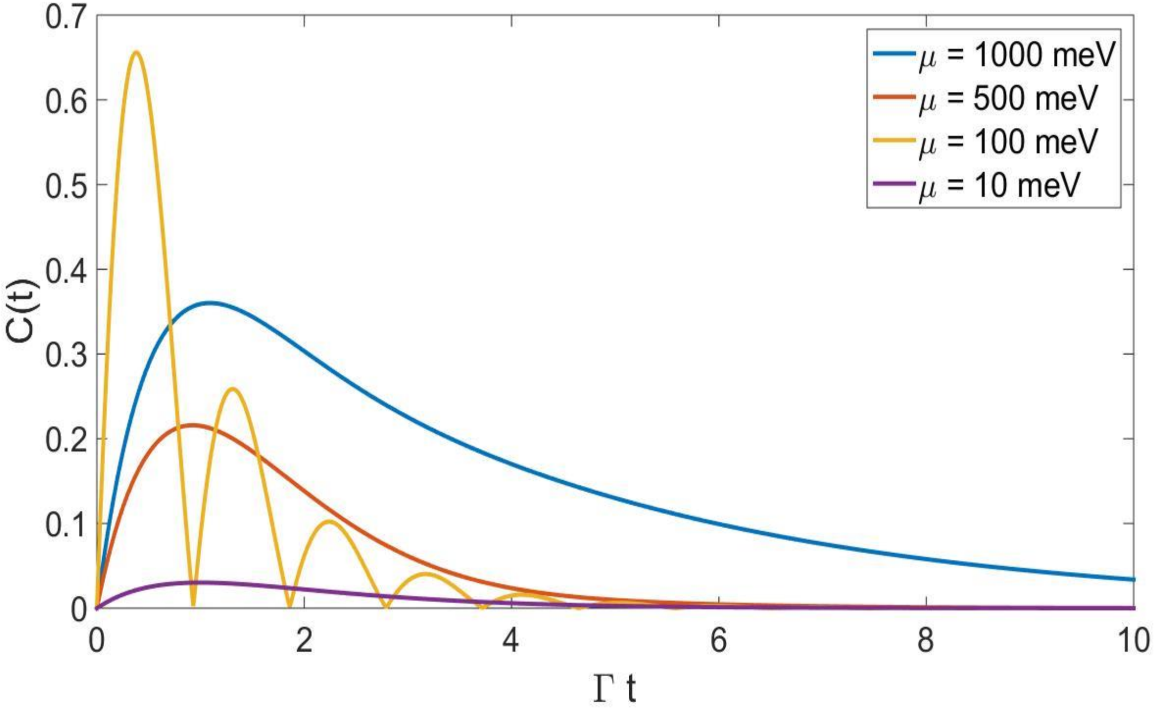




Graphene-Mediated Entanglement

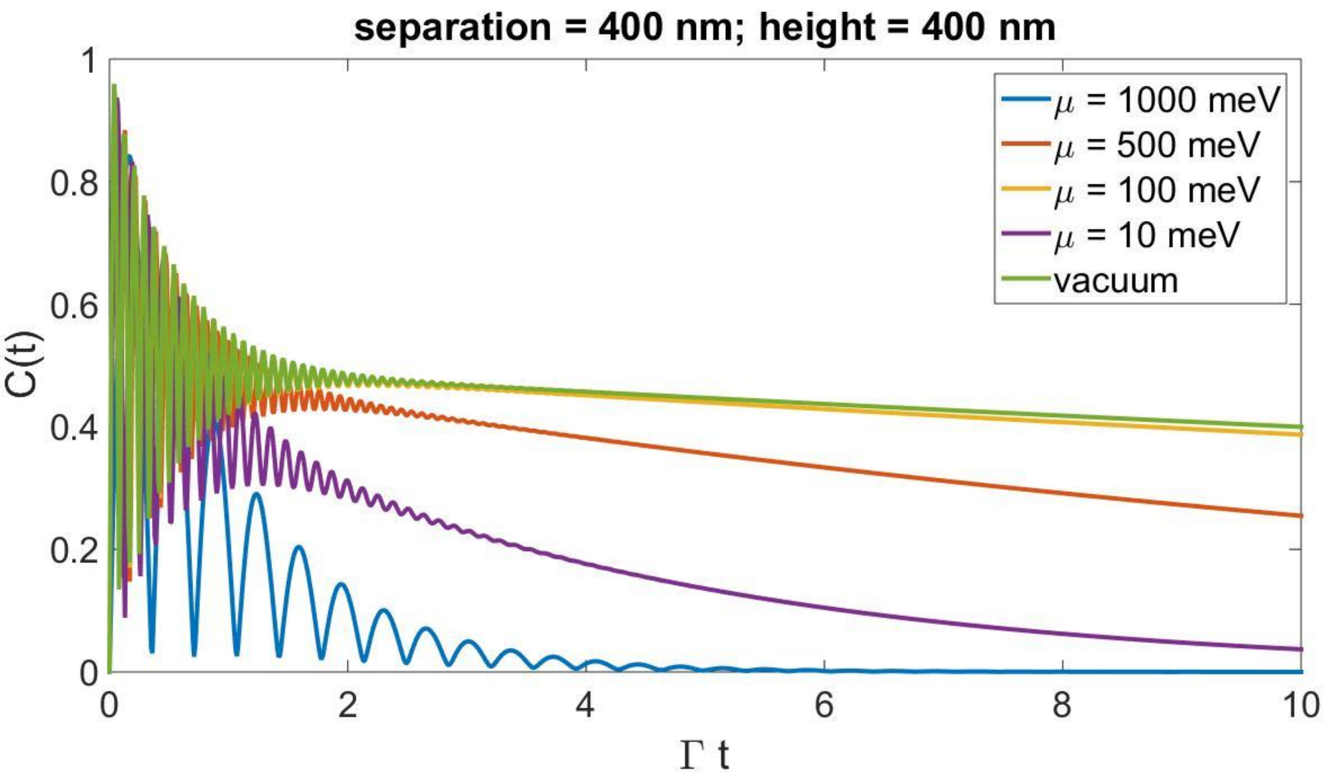
- However, we have found that graphene is not useful for long-distance entanglement
  - The graphene SPP is very tightly-confined to the surface, and, as a result, λ_{SPP} is too small ($\lambda_{SPP} \sim \lambda_0/10$ to $\lambda_0/100$) for long-distance propagation.
- Graphene does seem useful for **control** of entanglement.

Transient Entanglement via Spontaneous Emission over Graphene



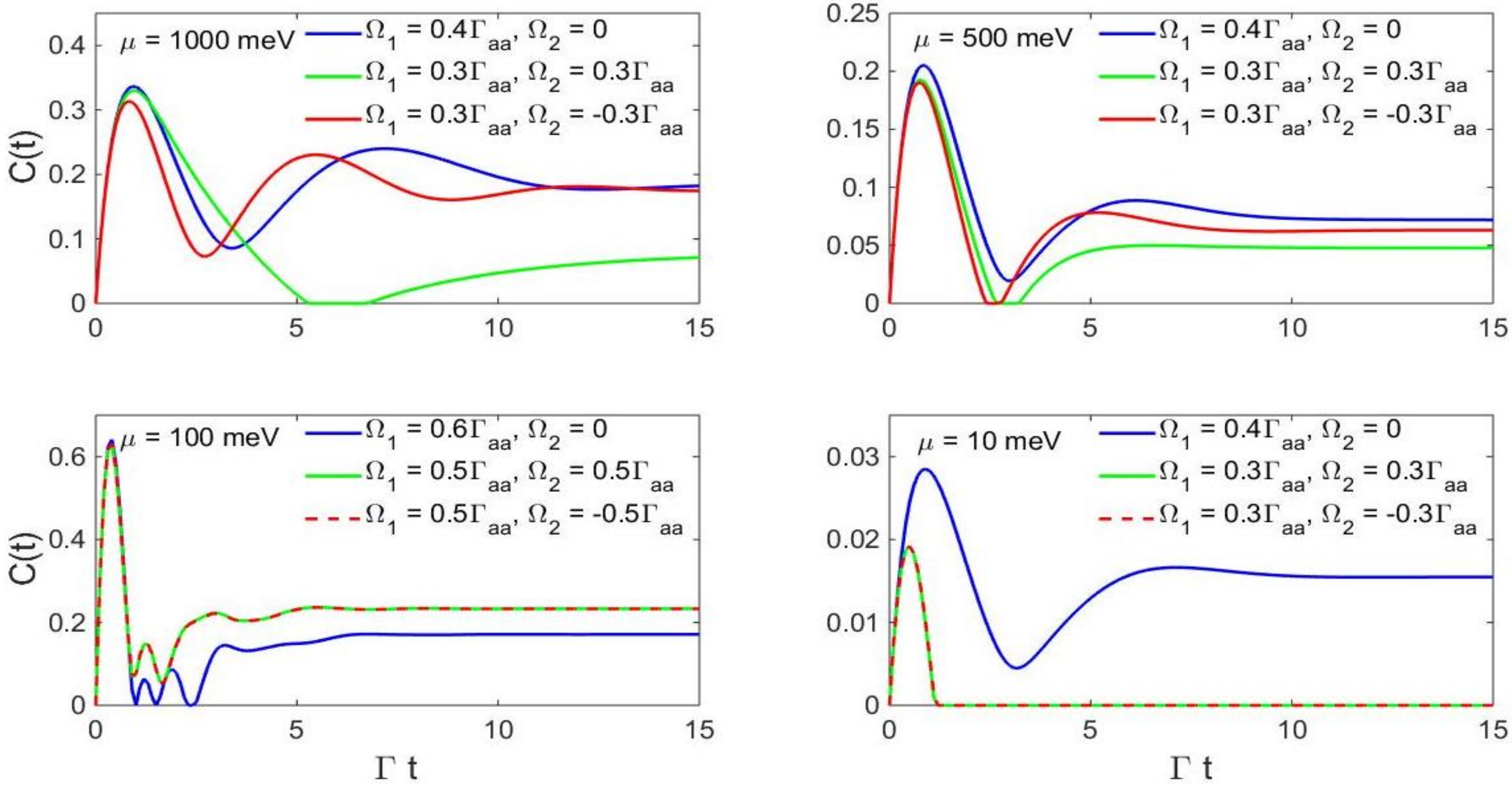
Transient entanglement between two quantum emitters placed at a distance 20 nm above graphene layer. Separation between emitters is equal to 100 nm. Frequency of the emitter dipole transition 40 THz.

Control Via Graphene Bias of Long-Lived Transient Entanglement



Transient entanglement between two quantum emitters placed at a distance 400 nm above a graphene layer. Separation between emitters is equal to 400 nm. Frequency of the emitter dipole transition is 40 THz.

Steady State Entanglement via External Pumping



Entanglement between two quantum emitters placed above graphene layer, which are pumped by external electromagnetic fields of intensities

$\Omega_j = \mathbf{d} \cdot \mathbf{E}_j/\hbar$ (the effective Rabi frequency of the pump)

Conclusions

- Graphene is a promising material for quantum applications.
- Tunability of the graphene conductivity, and subsequent affects on SPPs, is a principle motivation for various applications related to entanglement of quantum systems.

Thank You!

Τηανκ Ψου!