
Nathawut Homsup

Department of Electrical Engineering and Computer Science, Pennsylvania State University, University Park, PA, USA
Copyright

© The use of this work is restricted solely for academic purposes. The author of this work owns the copyright and no reproduction in any form is permitted without written permission by the authors.
Abstract

Impedance matching networks have wide range of application in the field of communication. Recent survey of literatures show different design procedures to realize its design and enhance its performance. This paper evaluated four nature-inspired optimization algorithm and introduces a new optimization algorithm, Modified Firefly Algorithm (MFA), which improves performances of the Standard Firefly Algorithm (SFA) in many ways. MFA was used to design both lossless and lossy impedance matching network and yielded results that were comparable to other optimization algorithms in literatures.

Index Terms: Global Optimization Method, Impedance Matching Network, Firefly Algorithm.
Biography

Nathawut Homsup received B.S. degree in Electrical Engineering from Chulalongkorn University, Bangkok, Thailand, in 2009, M.S. and Ph.D. degrees in Electrical Engineering from Pennsylvania State University, University Park, PA, USA, in 2011, and 2016, respectively. During M.S. degree studies, his research topics specialized in Digital Communication, Information Theory, and Error Correcting Code. During his Ph.D. degree studies, he worked for the Antenna and Radio Engineering Lab with researches related to Antenna Modelling and Design, Radar Remote Sensing, and Optimization Theory. His doctoral dissertation concerned Global Optimization Algorithms for Impedance Matching Network Designs.

From 2017, he has worked as a Postdoctoral Research Scholar for the Computational Electromagnetic Lab at University of Central Florida, Orlando, FL, USA with research topics including Computational Electromagnetic Theory, RF System Analysis and Antenna Design.
Antenna Impedance Matching

- Impedance matching is the connection of additional impedance elements to existing ones in order to achieve maximum power transfer, or to reduce reflection in the transmission line.
Performance of Impedance Matching Network (IMN)

- The performance of the IMN can be evaluated by a parameter called Transducer power gain (TPG).

- The other goal is to minimize the reflection coefficient ($\Gamma$). We could use the VSWR for this objective since it is directly related to the reflection.
Impedance Matching Techniques

Impedance matching Technique diagram
Outline

- Introduction to impedance matching
- Nature-inspired optimization algorithms
- Modified firefly algorithm (MFA)
- MFA on impedance matching problem
  - Lossless impedance matching
  - Lossy impedance matching
Optimization Algorithms

- Classical Methods
  - Newton’s Method
  - Gradient Descent Method

- Nature-Inspired Methods
  - Genetic Algorithm (GA)
  - Particle Swarm Optimization (PSO)
  - Ant Colony Optimization (ACO)
  - Firefly Algorithm (FA)
Genetic Algorithm (GA)

- Concept of GA
Swarm Intelligence Algorithm
Swarm Intelligence Algorithm

- Swarm intelligence algorithms are derived from the social behavior of animals that consist of a group of non-intelligent simple agents with no self-control to their behavior. The local interaction between the agents and their environment lead to the swarm behavior that is new to the individual agents.

- Examples of swarm intelligence algorithms are PSO, ACO, and FA.
Particle Swarm Optimization (PSO)

- **Concept of PSO**

\[
v_i = v_i + c_1 \cdot \text{rand} \cdot (p_{\text{best}} - x_i) + c_2 \cdot \text{rand} \cdot (g_{\text{best}} - x_i)
\]

\[
x_i = x_i + v_i
\]
Ant Colony Optimization (ACO)
Firefly Algorithm (FA)

- Attraction of a firefly to another is proportional to the Light intensity.

Light intensity

\[ I(r) = \frac{I_s}{r^2} \]
Lossless Impedance Matching

- The Pi3 network is shown below

This network was used for an impedance matching network. The load impedance can be obtained from measurement of a 80-meter inverted-V dipole antenna between frequencies 3.5-3.85 MHz. In simulation the antenna load data is from a record and a source resistance is 50 ohms.
Lossless Impedance Matching

- **Transducer Power Gain**

\[
TPG = 1 - |\Gamma|^2
\]

- \( \Gamma \) is the reflection coefficient, such that

\[
\Gamma = \frac{Z_{in} - Z_S}{Z_{in} + Z_S}
\]

\[
VSWR = \frac{1 + \Gamma}{1 - \Gamma}
\]

where \( Z_{in} \) is the input impedance seen by the generator at the equalizer input terminal.
Optimizers

- Global Optimization Toolbox from the MATLAB
  - Simulated Annealing (SA)
  - Pattern Search (PS)
  - Genetic Algorithm (GA)

- MATLAB codes of Nature-Inspired Optimization Methods
  - Genetic Algorithm (GA)
  - Particle Swarm Optimization (PSO)
  - Ant Colony Optimization (ACO)
  - Firefly Algorithm (FA)
Results – Global Optimization Toolbox

Pattern search

Simulated Annealing

GA with population = 20

GA with population = 200
Results – Nature-Inspired Methods

GA

PSO

ACO

FA
# Results - Comparison

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>PS</th>
<th>SA</th>
<th>GA</th>
<th>Population</th>
<th>N/A</th>
<th>N/A</th>
<th>20</th>
<th>200</th>
<th>20</th>
<th>150</th>
<th>70</th>
<th>150</th>
<th>70</th>
<th>150</th>
<th>70</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loops</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lowest max VSWR</td>
<td>1.4758</td>
<td>1.8664</td>
<td>2.0386</td>
<td>1.9199</td>
<td>1.3386</td>
<td>1.3386</td>
<td>1.3386</td>
<td>1.3386</td>
<td>1.3386</td>
<td>1.3386</td>
<td>1.3386</td>
<td>1.3386</td>
<td>1.3386</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 % error VSWR&lt;1.352</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>981</td>
<td>995</td>
<td>512</td>
<td>807</td>
<td>521</td>
<td>833</td>
<td>905</td>
<td>907</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 % error VSWR&lt;1.365</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>997</td>
<td>1000</td>
<td>618</td>
<td>860</td>
<td>631</td>
<td>884</td>
<td>985</td>
<td>983</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 % error VSWR&lt;1.379</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1000</td>
<td>1000</td>
<td>984</td>
<td>990</td>
<td>692</td>
<td>901</td>
<td>998</td>
<td>998</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time/sec</td>
<td>11.00</td>
<td>154.00</td>
<td>29.00</td>
<td>186.00</td>
<td>1.16</td>
<td>1.11</td>
<td>1.11</td>
<td>1.17</td>
<td>1.16</td>
<td>1.14</td>
<td>1.14</td>
<td>1.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Outline

- Introduction to impedance matching
- Nature-inspired optimization algorithms
  - Modified firefly algorithm (MFA)
- MFA on impedance matching problem
  - Lossless impedance matching
  - Lossy impedance matching
Standard Firefly Algorithm (SFA)

- Light intensity $I(r)$ is light intensity at a distance $r$ from the light source
  \[ I(r) = I_0 e^{-\gamma r^2} \]
  where $I_0$ is original light intensity, and $\gamma$ is the light absorption coefficient
- The attractiveness of firefly $j$ to firefly $i$ is defined as
  \[ \beta_{ij} = \beta_0 e^{-\gamma r_{ij}^2} \]
  where $\beta_0$ is the attractiveness at $r_{ij} = 0$
- The distance between firefly $j$ and firefly $i$ is defined as
  \[ r_{ij} = |X_i - X_j| = \sqrt{\sum_{k=1}^{d} (X_{i,k} - X_{j,k})^2} \]
Standard Firefly Algorithm (SFA)

\[ X_i^{t+1} = X_i^t + \beta_0 e^{-\gamma r_{ij}^2} (X_j^t - X_i^t) + \alpha \varepsilon_i \]

- First term is a position of firefly \( i \) at time \( t \)
- Second term is the exploitation or local search, that is a movement of firefly \( i \) toward firefly \( j \) base on their attractiveness and distance
- Third term is the exploration or randomness. \( \varepsilon_i \) can be specified as a random number generated with a value between 0 and 1
- \( \gamma \) and \( \alpha \) are very important parameters.
  - If \( \gamma \) approaches zero (\( \gamma \to 0 \)), the attractiveness is constant, \( \beta_{ij} = \beta_0 \), which means that every firefly can be seen in any position.
  - If the \( \gamma \) is nearing infinity or is too large (\( \gamma \to \infty \)), the attractiveness \( \beta_{ij} = 0 \) and the firefly movements become random
  - \( \alpha \) is a randomness parameter
Modified Firefly Algorithm (MFA)

The MFA improves the SFA in three points:

1) Scaling a computation domain into a unit hypercube
2) Having a realistic attractiveness between the fireflies
3) Including a new form of absorption coefficient and a randomness parameter
MFA – Scaling Domain

- The computation domain is scaled into a unit hypercube $u^d$ where $0 \leq u \leq 1$, and $d$ is the dimension of the searching domain.

$$X_{old} = (Upper\ Bound - Lower\ Bound) \cdot X_{new} + Lower\ Bound$$

$$X_{new} = \frac{(X_{old} - Lower\ Bound)}{(Upper\ Bound - Lower\ Bound)}$$
MFA – Modified Attractiveness

- The attractiveness of firefly $i$ toward firefly $j$, $\beta_{ij}$ is modified as
  $\beta_{ij} = \beta_0 I_j e^{-\gamma r^2}$

- Thus,
  $r = r_{ij}/\sqrt{d}$

- where $I_j = (f(x_j) - f_{\text{min}})/f_{\text{max}} - f_{\text{min}}$
- $f_{\text{max}}$ is the maximum of $f(X_j)$, $j = 1, \ldots, N$
- $f_{\text{min}}$ is the minimum of $f(X_j)$, $j = 1, \ldots, N$
MFA – Modified Attractiveness

- The firefly $i$ is attracted towards the more attractive firefly $j$, the movement is defined as

$$X_{i}^{t+1} = X_{i}^{t} + \beta_{ij}(X_{j}^{t} - X_{i}^{t}) + \alpha \varepsilon_{i}$$

- Thus,

$$X_{i}^{t+1} = X_{i}^{t} + \beta_{0}I_{j}e^{-\gamma r^{2}}(X_{j}^{t} - X_{i}^{t}) + \alpha \varepsilon_{i}$$

- The typical value of $\beta_{0}$ is 1.
MFA – New Absorption Coefficient

- We want the exploration to decrease, but exploitation to increase as the number of iterations increase. Thus, the parameters $\gamma$ and $\alpha$ should be gradually reduced. Both parameters can be formulated as

$$\gamma = \gamma_0 \cdot (\theta_1)^t \quad \quad \alpha = \alpha_0 \cdot (\theta_2)^t$$

where $0 < \theta_1, \theta_2 < 1$. $\gamma_0$, and $\alpha_0$ are the initial value of $\gamma$, and $\alpha$, respectively.
MFA – New Absorption Coefficient

- In practice, we want to have $\gamma$ and $\alpha$ to be very small values at the end of iteration. For example, let

$$\gamma = \gamma_0 \cdot (\theta_1)^t = \varepsilon_1 \quad \text{and} \quad \alpha = \alpha_0 \cdot (\theta_2)^t = \varepsilon_2$$

at $t = \text{Max\_Iteration}$, thus,

$$\theta_1 = e^{(\log(\varepsilon_1/\gamma_0)/\text{Max\_Iteration})} \quad \text{and} \quad \theta_2 = e^{(\log(\varepsilon_2/\alpha_0)/\text{Max\_Iteration})}$$

- For example,

  If $\varepsilon_1 = 0.001$, $\text{Max\_iteration} = 50$, $\gamma_0 = 25$, we will have $\theta_1 = 0.8167$

  If $\varepsilon_2 = 0.001$, $\alpha_0 = 0.03$, we will have $\theta_2 = 0.8922$
For simplicity, one can use $\beta_0 = 1$; therefore, the two parameters to be tuned are $\gamma_0 > 0$ and $0 < \alpha_0 < 1$.

It can be seen that $\gamma$ controls the scaling and it indicates how fast amplitude can fall, while $\alpha$ controls the randomness.

Consider $e^{-\gamma_0 r^2}$, we define a characteristic length, $\Gamma$, as the value of $r$ such that $e^{-\gamma_0 r^2} = e^{-1}$. This means that

$$\gamma_0 = 1/\Gamma^2,$$

where $0 < \Gamma < 1$.

Then, if we choose $\Gamma$, so that $\gamma_0$ satisfied the equation above
Define the formulas that will be used to compute the number of iterations ($Max_it$) and number of the firefly population ($N$) as follows:

\[
Max_it = 20 \cdot d + 10
\]

\[
N = 5 \cdot d + 40
\]

where $d$ is the dimension of the problem.
Ackley Function

The formula of Ackley Function is

\[ f(x, y) = -20 \exp(-0.2\sqrt{0.5(x^2 + y^2)}) - \exp(0.5(\cos(2\pi x) + \cos(2\pi y))) + e + 20 \]

The search space is restricted to \(-5 \leq x, y \leq 5\). The global minimum of this function is equal to zero, attained at (0,0).
Ackley Function

- The formula of Ackley Function is

Locations of fireflies after 10 iterations

Final locations of fireflies after 50 iterations

Comparison of the performance of SFA and MFA for minimization of the Ackley function with $\gamma_0 = 25$, and $\alpha_0 = 0.02$
Sphere Function

- The formula of sphere function is

\[ f(x, y) = x^2 + y^2 \]

The search space is restricted to \(-5 \leq x, y \leq 5\). The global minimum of this function is equal to zero, attained at (0,0).
Sphere Function

- The formula of sphere function is

Locations of fireflies after 10 iterations

Final locations of fireflies after 50 iterations

Comparison of the performance of SFA and MFA with $\gamma_0 = 150$, and $\alpha_0 = 0.3$
Rastrigin Function

- The formula of Rastrigin function is

\[ f(x, y) = 20 + (x^2 - 10 \cdot \cos(2 \cdot \pi x)) + (y^2 - 10 \cos(2 \cdot \pi y)) \]

The search space is restricted to \( -5 \leq x, y \leq 5 \). The global minimum of this function is equal to zero, attained at (0,0).
Rastrigin Function

- The formula of Rastrigin function is

Locations of fireflies after 10 iterations

Final locations of fireflies after 50 iterations, converging into (0,0)

Comparison of the performance of SFA and MFA with $\gamma_0 = 30$, $\alpha_0 = 0.1$, and $Max\_it = 500$
Michalewicz Function

- The formula of Michalewicz function is

$$f(x, y) = \sin(x) \cdot \left(\sin\left(\frac{x^2}{\pi}\right)^{2m} + \sin(y) \cdot \left(\sin\left(\frac{2y^2}{\pi}\right)^{2m}\right)\right)$$

The search space is restricted to $-5 \leq x, y \leq 5$. The global minimum of this function is equal to -1.80, attained at $(2.20, 1.57)$. 

$$-f(x, y)$$
Michalewicz Function

The formula of Michalewicz function is

Locations of fireflies after 10 iterations

Final locations of fireflies after 50 iterations

Comparison of the performance of SFA and MFA with $\gamma_0 = 20$, and $\alpha_0 = 0.02$
Outline

- Introduction to impedance matching
- Nature-inspired optimization algorithms
- Modified firefly algorithm (MFA)
- MFA on impedance matching problem
  - Lossless impedance matching
  - Lossy impedance matching
Lossless Impedance Matching

- The Pi3 network is shown below

- This network was used for an impedance matching network. The load impedance can be obtained from measurement of a 80-meter inverted-V dipole antenna between frequencies 3.5-3.85 MHz. In simulation the antenna load data is from a record (A15.txt) and a source resistance is 50 ohms.
**Lossless Impedance Matching**

- **Transducer Power Gain**
  \[ TPG = 1 - |\Gamma|^2 \]

- \( \Gamma \) is the reflection coefficient, such that
  \[ \Gamma = \frac{(Z_{in} - Z_S)}{(Z_{in} + Z_S)} \]

  where \( Z_{in} \) is the input impedance seen by the generator at the equalizer input terminal.

  \[ VSWR = \frac{1 + \Gamma}{1 - \Gamma} \]

- With \( \gamma_0 = 100 \), \( \alpha_0 = 0.6 \), population = 55, and Max_iteration = 70, an average execution time is 30.72 sec. the search space is as follows:

  \[ 1 \text{nF} \leq C_1 \leq 8 \text{nF} \quad , \quad 1 \text{nF} \leq C_2 \leq 8 \text{nF} \quad , \text{and} \quad 0.1 \mu H \leq L \leq 3 \mu H \]
Lossless Impedance Matching

- The optimized component values are obtained as follows:

\[ C_1 = 4.09 \, nF, \quad C_2 = 7.83 \, nF, \quad \text{and} \quad L = 0.6818 \, \mu H \]

- VSWR of lossless Pi3 network with Min_VSWR = 1.1107, and Max_VSWR = 1.3386
Outline

- Introduction to impedance matching
- Nature-inspired optimization algorithms
- Modified firefly algorithm (MFA)
- MFA on impedance matching problem
  - Lossless impedance matching
  - Lossy impedance matching
Lossy Impedance Matching

- By adding loss, more bandwidth may be acquired at a cost in the transducer power gain (TPG) of matching networks.
Lossy Impedance Matching - Topology

- Lossy network topology with one resistor

- Lossy network topology with two resistors

- Lossy network topology with three resistors
Lossy Impedance Matching

- TPG can be calculated from

\[ TPG = \frac{4 \text{Re}\{Z_S\} \text{Re}\{Z_L\} |z_{21}|^2}{|(Z_S + z_{11})(Z_L + z_{22}) - z_{12}z_{21}|^2} \]

- where \( Z_S \) is the source impedance, \( Z_L \) is the load impedance. \( z_{11}, z_{12}, z_{21}, \) and \( z_{22} \) are values of Z-parameter of the matching network.
Lossy Impedance Matching

- Z-parameter of the matching network, that can be obtained from

\[ Y_{c1} = j\omega C_1, \quad X_{c2} = \frac{1}{j\omega C_2}, \quad Y_{L1} = \frac{1}{j\omega L_1} \]

\[ AR = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}, \quad AC_1 = \begin{bmatrix} 1 & 0 \\ Y_{c1} & 1 \end{bmatrix}, \quad AC_2 = \begin{bmatrix} 1 & X_{c2} \\ 0 & 1 \end{bmatrix}, \quad AL_1 = \begin{bmatrix} 1 & 0 \\ Y_{L1} & 1 \end{bmatrix} \]

\[ \begin{bmatrix} A & B \\ C & D \end{bmatrix} = AR \ast AC_1 \ast AC_2 \ast AL_1 \]

\[ \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{C} \begin{bmatrix} A & (AD - BC) \\ 1 & D \end{bmatrix} \]
Lossy Impedance Matching

- We can find a reflection coefficient (\( \Gamma \)) as following steps

\[
Z_A = \frac{(j\omega L \cdot Z_L)}{(j\omega L + Z_L)}
\]

\[
Z_B = \frac{1}{j\omega C_2} + Z_A
\]

\[
Z_C = \frac{1}{j\omega C_1} \cdot Z_B \bigg/ \left( \frac{1}{j\omega C_1} + Z_B \right)
\]

\[
Z_{in} = R + Z_C
\]

\[
\Gamma = \frac{Z_{in} - Z_S}{Z_{in} + Z_S}
\]

- The source resistance, \( Z_S = 50 \ \Omega \), and load data are obtained from a record (A15.txt).
Lossy Impedance Matching

- Two objectives are $J_1 = |\Gamma|$, and $J_2 = 1 - TPG$
- The final objective function ($F$) to be minimized, consisting of two objectives, is
  
  $$ F = (\lambda J_1 + (1 - \lambda)J_2) $$

  Thus,

  $$ F = (\lambda|\Gamma| + (1 - \lambda)(1 - TPG)) $$

  where $0 \leq \lambda \leq 1$, in frequency range of 3.5-3.85 MHz.

- The MFA program run with $\gamma_0 = 100$, $\alpha_0 = 0.6$, population = 200, and Max_iterations = 60. Each program run has average execution time of 4.30 minutes. The search space used in the optimization is as follows:

  $40 \leq C_1 \leq 2000 \, pF$, $40 \leq C_3 \leq 2000 \, pF$, $40 \leq L_2 \leq 3000 \, nH$, and $1 \leq R \leq 1000 \, \Omega$
Lossy Impedance Matching

The optimized component values are shown below

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( c_1/\rho F )</th>
<th>( c_2/\rho F )</th>
<th>( L/nH )</th>
<th>( R/\Omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>924.4</td>
<td>1670.7</td>
<td>2681.6</td>
<td>29.4</td>
</tr>
<tr>
<td>0.1</td>
<td>1689.0</td>
<td>1786.2</td>
<td>1930.8</td>
<td>13.8</td>
</tr>
<tr>
<td>0.2</td>
<td>1397.4</td>
<td>1908.2</td>
<td>2054.0</td>
<td>23.6</td>
</tr>
<tr>
<td>0.3</td>
<td>1199.7</td>
<td>1777.2</td>
<td>2258.2</td>
<td>8.5</td>
</tr>
<tr>
<td>0.4</td>
<td>1294.5</td>
<td>1572.3</td>
<td>2237.4</td>
<td>28.1</td>
</tr>
<tr>
<td>0.5</td>
<td>886.5</td>
<td>1442.8</td>
<td>2603.1</td>
<td>18.8</td>
</tr>
<tr>
<td>0.6</td>
<td>551.3</td>
<td>1100.9</td>
<td>2824.6</td>
<td>28.0</td>
</tr>
<tr>
<td>0.7</td>
<td>1443.0</td>
<td>1509.4</td>
<td>2009.0</td>
<td>28.7</td>
</tr>
<tr>
<td>0.8</td>
<td>928.8</td>
<td>1285.5</td>
<td>1692.6</td>
<td>40.8</td>
</tr>
<tr>
<td>0.9</td>
<td>1628.5</td>
<td>1503.0</td>
<td>1395.4</td>
<td>42.4</td>
</tr>
<tr>
<td>1.0</td>
<td>1522.0</td>
<td>1581.5</td>
<td>1098.5</td>
<td>45.9</td>
</tr>
</tbody>
</table>
Lossy Impedance Matching

TPG of lossy Pi3 network for $\lambda=0, 0.1, \ldots, 1$
Lossy Impedance Matching

VSWR of lossy Pi3 network for $\lambda=0, 0.1, \ldots, 1$
Lossy Impedance Matching

- Minimum VSWR, maximum VSWR, minimum TPG, and maximum TPG result of lossy Pi3 network for weight $\lambda = 0, 0.1, ..., 1$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>Min_VSWR</th>
<th>Max_VSWR</th>
<th>Min_TPG</th>
<th>Max_TPG</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.7101</td>
<td>2.6209</td>
<td>0.9543</td>
<td>0.9600</td>
</tr>
<tr>
<td>0.1</td>
<td>1.2693</td>
<td>1.7201</td>
<td>0.9350</td>
<td>0.9446</td>
</tr>
<tr>
<td>0.2</td>
<td>1.1151</td>
<td>1.5337</td>
<td>0.9509</td>
<td>0.9894</td>
</tr>
<tr>
<td>0.3</td>
<td>1.1072</td>
<td>1.4748</td>
<td>0.9389</td>
<td>0.9980</td>
</tr>
<tr>
<td>0.4</td>
<td>1.1382</td>
<td>1.4733</td>
<td>0.9112</td>
<td>0.9916</td>
</tr>
<tr>
<td>0.5</td>
<td>1.1056</td>
<td>1.4696</td>
<td>0.8494</td>
<td>0.9749</td>
</tr>
<tr>
<td>0.6</td>
<td>1.1169</td>
<td>1.3468</td>
<td>0.9362</td>
<td>0.9990</td>
</tr>
<tr>
<td>0.7</td>
<td>1.0585</td>
<td>1.3006</td>
<td>0.7301</td>
<td>0.9256</td>
</tr>
<tr>
<td>0.8</td>
<td>1.1006</td>
<td>1.2607</td>
<td>0.7167</td>
<td>0.9197</td>
</tr>
<tr>
<td>0.9</td>
<td>1.0318</td>
<td>1.0908</td>
<td>0.2916</td>
<td>0.4647</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0023</td>
<td>1.0080</td>
<td>0.0234</td>
<td>0.0429</td>
</tr>
</tbody>
</table>
Reference


