Orbital Angular Momentum Generation Using a Bi-Layered Complementary Metasurface with a High Conversion Efficiency

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Abstract: Electromagnetic (EM) waves with helical wavefront carry orbital angular momentum (OAM), which is associated with the azimuthal phase of the complex electric field. OAM is a new degree of freedom in EM waves and is promising for channel multiplexing in communication system. Although the OAM-carrying EM wave attracts more and more attention, the method of OAM generation at microwave frequencies still faces challenges, such as efficiency and simulation time. In this work, by using the circuit theory and equivalence principle, we build two simplified models, one for a single scatter and one for the whole metasurface to predict their EM responses. Both of the models significantly simplify the design procedure and reduce the simulation time. In this paper, we propose an ultrathin complementary metasurface that converts a left-handed (right-handed) circularly polarized plane wave without OAM to a right-handed (left-handed) circularly polarized wave with OAM of arbitrary orders and a high transmission efficiency can be achieved.

Keywords: Orbital angular momentum, ultrathin complementary metasurface, circuit theory, equivalence principle, Babinet’s principle.

References:


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- Proposed ultra-thin complementary metasurface
  - Theoretical derivation
  - Conditions for ideal OAM generation
- Modelling
  - Equivalent circuit model
  - Magnetic dipole approximation
- Efficiency
- Conclusions
INTRODUCTION

- Angular momentum of light
  - Spin angular momentum (SAM), single photon, $\pm h$, 0
  - **Orbital angular momentum (OAM):** optical vortex beam

- Applications
  - Communication: radio, optical, quantum
  - Optical tweezers
HISTORY OF OAM

- Laguerre-Gaussian Modes\textsuperscript{[1]}

\[ LG_{p\ell} = \sqrt{\frac{2p!}{\pi (p + |\ell|)!}} \frac{1}{w(z)} \left[ \frac{r \sqrt{2}}{w(z)} \right]^{|\ell|} \exp \left[ -\frac{r^2}{w^2(z)} \right] \frac{L_{p}^{|\ell|}}{w(z)} \left( \frac{2r^2}{w^2(z)} \right) \exp[i\ell\phi] \times \exp \left[ \frac{ik_0r^2z}{2(z^2 + z_R^2)} \right] \exp \left[ -i(2p + |\ell| + 1)\tan^{-1} \left( \frac{z}{z_R} \right) \right] \]

REVIEW OF CURRENT WORK

- Working principles
  - Spiral phase plate (SPP)
    \[ e^{-i\ell d} \rightarrow e^{-i\ell \varphi} \]
  - Spin-Orbit interactions

- Existing prototypes

Metasurface I \(^{[1]}\)

Metasurface II \(^{[2]}\)

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THEORETICAL DERIVATION

- **Jones vector**
  - Describes the polarization state of light

\[
E_i(r, t) = \begin{pmatrix} E_{ix}^i \\ E_{iy}^i \end{pmatrix} e^{i(kz - \omega t)} \quad E_s(r, t) = \begin{pmatrix} E_{sx}^s \\ E_{sy}^s \end{pmatrix} e^{i(kz - \omega t)}
\]

\(E_{x,y}^i\) and \(E_{x,y}^s\) are Jones vectors for incident and scattered waves.

- **Jones matrix**
  - Models an optical element / scatterer

\[
\begin{pmatrix} E_{sx}^s \\ E_{sy}^s \end{pmatrix} = \begin{pmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{pmatrix} \begin{pmatrix} E_{ix}^i \\ E_{iy}^i \end{pmatrix} = \mathbf{J} \begin{pmatrix} E_{ix}^i \\ E_{iy}^i \end{pmatrix}
\]
THEORETICAL DERIVATION (CONT.)

- **Scatterer under rotation**
  \[
  \mathbf{J} = \begin{pmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{pmatrix}
  \]
  \[
  \mathbf{J}(\alpha) = R(-\alpha) \mathbf{J} R(\alpha)
  \]
  \[
  R(\alpha) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}
  \]

- **Special form**
  - \( J_{yy} = -J_{xx} \) & \( J_{yx} = J_{xy} \)
  \[
  \mathbf{J}(\alpha) = \begin{pmatrix} J_{xx} \cos(2\alpha) - J_{xy} \sin(2\alpha) & J_{xx} \sin(2\alpha) + J_{xy} \cos(2\alpha) \\ J_{xx} \sin(2\alpha) + J_{xy} \cos(2\alpha) & J_{xy} \sin(2\alpha) - J_{xx} \cos(2\alpha) \end{pmatrix}
  \]

- **Circular basis**
  \[
  \mathbf{J}_c(\alpha) = \begin{pmatrix} 0 & e^{-2i\alpha}(J_{xx} - iJ_{xy}) \\ e^{2i\alpha}(J_{xx} + iJ_{xy}) & 0 \end{pmatrix}
  \]
  \( e^{\pm 2i\alpha} \), geometric phase
CONDITIONS FOR IDEAL OAM GENERATION

- Scatterers with spatially varying rotation angle \( \alpha \) according to its azimuthal location angle \( \phi \)
  - OAM order of \( l = 2\alpha / \phi \)

- Perfect conversion (100%)
  - \( J_{xx} \) and \( J_{yy} \) have unit amplitude and 180° phase difference
  - \( J_{xy} = J_{yx} = 0 \)

\[
J_{yy} = -J_{xx} \quad \text{and} \quad J_{yx} = J_{xy} \\
\mathbf{J}_c(\alpha) = \begin{pmatrix}
0 & e^{-2i\alpha}(J_{xx} - iJ_{xy}) \\
e^{2i\alpha}(J_{xx} + iJ_{xy}) & 0
\end{pmatrix}
\]
SCATTERER DESIGN

- **Objective**
  - Scatterer: transmission type
  - Requirements: \( \text{mag}(T_{yy}) = \text{mag}(T_{xx}) \), \( \text{phase}(T_{yy}) - \text{phase}(T_{xx}) = 180° \) and \( T_{yx} = T_{xy} = 0 \)

- **Complementary frequency selective surface (FSS)**
  - One-layer split ring resonator
  - Equivalent circuit model

- **Problem**
  - Cannot achieve the phase requirement and high transmission simultaneously

\[ \begin{align*}
Z_0 & \quad C_0 \quad L_0 \quad Z_0 \\
\end{align*} \]
SCATTERER DESIGN (CONT.)

- Bi-layer complementary split ring resonators (CSRRs)

\[ \text{y polarization} \]

\[ \text{x polarization} \]

- Equivalent circuit model
  - Can achieve the phase requirement with high transmission simultaneously
SCATTERER SIMULATION

- Simulated transmission coefficients of the proposed scatterer
  - @ Designed frequency (17.85 GHz)
    - Magnitudes of the co-polarized transmission coefficients are 0.91
    - Phase difference is 180°
    - 81% right-to-left (left-to-right) circular polarization conversion efficiency

Dielectric substrate: F4B220, $\varepsilon_r = 2.2$, $h = 0.8$ mm
Geometric parameters: The period of the unit cell is $7 \times 7$ mm$^2$. Side lengths of the two types of square CSRRs are $a_1 = 5.2$ mm and $a_s = 3.9$ mm. The length of the complementary gap is $g = 0.2$ mm. The width of the slots is $t = 0.2$ mm.
METASURFACE DESIGN

- Geometry of the metasurfaces (top view)

\[
\text{OAM of order } l = \frac{2\alpha}{\varphi} = 2
\]

\[
\text{OAM of order } l = \frac{2\alpha}{\varphi} = 4
\]

- Simulation approaches
  - Equivalent magnetic dipoles sources, faster
  - Simulation software

\[\alpha, \text{ rotation angle}\]
\[\varphi, \text{ azimuthal angle}\]
METASURFACE DESIGN (CONT.)

- Magnetic dipole approximation
  - Equivalent magnetic point source
  - Green’s function
  - Excitation: right-handed circularly polarized wave

\[
E(r) = 2 \int_V \overline{G}_m(r, r') \cdot M(r') \, dr' = 2 \int_V \nabla g(r, r') \times M(r') \, dr'
\]
METASURFACE CALCULATION RESULTS

- Calculated field patterns at a transverse plane of $z = 0.6\lambda_0$

**OAM of order 2**

**OAM of order 4**
METASURFACE SIMULATION RESULTS

- Simulated cross-circular polarized field patterns at a transverse plane of $z = 0.6\lambda_0$

**Amplitude distribution**

**Phase distribution**

**OAM of order 2**

**OAM of order 4**

*Excitation: right-handed circularly polarized Gaussian beam*
**Efficiency**

- Efficiency 81.8% (ratio of the energy carried by the OAM wave in reference to the total energy of the transmitted wave)
- Efficiency 15.2% (ratio of the energy carried by the OAM wave in reference to the total energy of the incident Gaussian beam)

\[ U = \oint_A \mathbf{S} \cdot d\mathbf{A} \]

- OAM of order 2
- Cross-circular polarization
- Co-circular polarization
Efficiency (the ratio of energy carried by the OAM wave in reference to the total energy of the incident Gaussian beam)

- 39%, 55% and 42% for Region 1, 2 and 3, respectively

Due to the truncation effect of the finite size of the metasurface, the periodicity along \( \theta \) and \( r \) directions is only preserved in Region 2.

By increasing the number of scatterers on the metasurface, the periodicity can be preserved better so that the efficiency can be improved.
CONCLUSIONS

- Orbital angular momentum (OAM), \( L = l\hbar \)
- An ultrathin complementary metasurface to generate OAM at microwave frequencies is proposed.
  - The circuit model is established to reveal the working physics and facilitate the design optimization
  - The whole metasurface is modelled by the equivalent magnetic dipole sources. The calculated results show a good agreement with the full-wave simulation results.
  - The efficiency of the proposed metasurface is carefully investigated. Thanks to the complementary design, a high transmission efficiency can be potentially achieved by increasing the number of the proposed scatterers.

Q & A

THANK YOU