Graphene Magnetoplasmonic Principles, Structures and Devices

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Abstract:
This paper recalls fundamentals of magnetoplasmons in magnetically biased graphene structures, describes their non-reciprocity and demonstrates their utilization as devices such as isolators and couplers. A multi-scale multi-physics structure, using a magnetic nanowire membrane as integrable magnetic bias, with applications to Faraday rotators and integrable nonreciprocal plasmonic components, is presented.

Keywords: graphene, plasmonics, magnetoplasmon, nonreciprocity, isolator, coupler.

References:

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Graphene Magnetoplasmonic Principles, Structures and Devices

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I. INTRODUCTION TO GRAPHENE MAGNETOPLASMONS

II. PRINCIPLES OF NON-RECIPROCITY

III. STRUCTURES AND DEVICES
   I. NONRECIPROCAL PHASE SHIFTER
   II. NON-RECIROCAL COUPLER
   III. MAGNETOPLASMONIC ISOLATORS
   IV. MAGNETIC SENSOR

❖ CONCLUSIONS & QUESTIONS
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❖ CONCLUSIONS & QUESTIONS
Surface Plasmons - Introduction

C. Neto et al., Rev. Mod. Phys. 81, 109 –January 2009

dielectric

metal

metal-dielectric interface

2D electron gas (2DEG)

dielectric

metal

dielectric

graphene:

- one atom thick material
- gapless energy band
- 2D electron system with high mobility
- tunability and ambipolarity
$B_0 = 0$
$\Im(\sigma) < 0 \implies \text{longitudinal (TM}_z\text{)}$

$\Im(\sigma) > 0 \implies \text{transverse (TE}_z\text{)}$

$B_0 > 0$
$\implies \text{hybrid mode}$
Plasmons in a Graphene Strip

- graphene strip without magnetic bias
- 2D eigenvalue problem
- finite difference frequency domain (FDFD)
- graphene: zero thickness conductive strip
- Kubo conductivity tensor
- infinite number of 2D bulk modes
- 2 degenerate edge modes

\[ B_0 = 0 \]

\[ n_s = 10^{13} \text{ cm}^{-2} \]
\[ \tau = 0.1 \text{ ps} \]
\[ w = 100 \text{ μm} \]
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❖ CONCLUSIONS & QUESTIONS
Splitting of the Edge Modes with a Magnetic Bias

\[ \sigma = \sigma_d - j\sigma_o \]

\[ \bar{\sigma} = \sigma_d(\hat{x}\hat{x} + \hat{z}\hat{z}) + \sigma_o(\hat{x}\hat{z} - \hat{z}\hat{x}) \]

Magnetically Biased Graphene Strip

Edge modes
- split in a magnetic field
- exhibit non-reciprocal properties

\[ n_s = 10^{13} \text{ cm}^{-2} \]
\[ \tau = 0.1 \text{ ps} \]
\[ w = 100 \mu\text{m} \]
\[ B = 1 \text{ T} \]
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Nonreciprocal Plasmonic Phase Shifter

Edge modes
- unsymmetric dispersions for opposite directions
- $\Rightarrow$ different phase shifts

![Diagram of nonreciprocal plasmonic phase shifter with edge modes labeled and graphs showing frequency vs. phase difference and frequency vs. attenuation for different sheet densities.](image-url)
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Nonreciprocal Magnetoplasmon Coupler

left strip:
\[ n_s = 8 \times 10^{12} \text{ cm}^{-2} \]
\[ \tau = 0.1 \text{ ps} \]
\[ w = 100 \mu\text{m} \]
\[ B = 1 \text{T} \]

right strip:
\[ n_s = 10^{13} \text{ cm}^{-2} \]
\[ \tau = 0.1 \text{ ps} \]
\[ w = 100 \mu\text{m} \]
\[ B = 1 \text{T} \]
Nonreciprocal Magnetoplasmon Coupler
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❖ CONCLUSIONS & QUESTIONS
Graphene Plasmonic Isolator

no magnetic bias:
- symmetric dispersion for opposite directions
P-N Junction Mode Forward Vs. Backward

\[ \sigma = \sigma_d + j\sigma_{on} \quad \text{forward} \quad \sigma = \sigma_d - j\sigma_{op} \]

\[ \sigma = \sigma_d - j\sigma_{on} \quad \text{backward} \quad \sigma = \sigma_d + j\sigma_{op} \]

- **n-doped**: \( \bar{\sigma} = \sigma_d(\hat{x}\hat{x} + \hat{z}\hat{z}) + \sigma_o(\hat{x}\hat{z} - \hat{z}\hat{x}) \)
- **p-doped**: \( \bar{\sigma} = \sigma_d(\hat{x}\hat{x} + \hat{z}\hat{z}) - \sigma_o(\hat{x}\hat{z} - \hat{z}\hat{x}) \)
with magnetic bias:
- unsymmetric dispersion for opposite directions
- mode 3 is localized at the p-n junction only
  in the forward direction
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❖ CONCLUSIONS & QUESTIONS
Electrically Doped Graphene PN Junction

\[ \int_{-w/2}^{w/2} \rho(x', y') G(x, y; x', y') dx' - E_0 x = 0, \]  
\[ -w/2 \leq x \leq w/2, \quad y = 0, \quad y' = 0, \]  
\[ G(x, y; x', y') = \frac{-1}{2\pi \epsilon_0} \ln \sqrt{(x' - x)^2 + (y' - y)^2} \]

2D Green function for Poisson equation

\[ w = 50 \mu m \]
\[ E_0 = 10^8 \text{ V/m} \]
Electrically Doped Plasmonic ISolator

$B \uparrow \Rightarrow$ the localized p-n junction mode propagates only in the forward direction

$w = 50 \, \mu m$
$E_0 = 10^8 \, V/m$
$B = 0.1 \, T$
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❖ CONCLUSIONS & QUESTIONS
TM (conventional) Surface Plasmons

Electric field

LHCP

RHCP

charges
Graphene TE Surface Plasmons

- LHCP
- RHCP
- Magnetic field
- Transverse currents
TE Plasmons Interaction with Ferrite Substrate

LHCP Forward
RHCP backward

Static magnetic field

Ferrite

magnetic field
Analysis 1

Electric fields
\[ E_1 = \left( E_{x1}, E_{y1}, E_{z1} \right) e^{-\alpha_1 y - ikz} \]
\[ E_2 = \left( E_{x2}, E_{y2}, E_{z2} \right) e^{+\alpha_2 y - ikz} \]

Magnetic fields
\[ H = \frac{-1}{i\omega\mu_0} \bar{\mu}^{-1} \cdot \nabla \times \mathbf{E} \]

Graphene sheet
\[ \varepsilon_1, \mu_1 \]

Ferrite
\[ \varepsilon_2, \bar{\mu}_2 \]
\[ \mu_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mu_{rd2} & -\mu_{ro2} \\ 0 & \mu_{ro2} & \mu_{rd2} \end{bmatrix} \]

Electric fields
\[ H_1 = \left[ \frac{i}{\mu_0 \mu_{r1} \omega} \left( iE_{y1} k e^{-\alpha_1 y - ikz} - E_{z1} \alpha_1 e^{-\alpha_1 y - ikz} \right) \right] \]
\[ \frac{E_{z1} k}{\mu_0 \mu_{r1} \omega} e^{-\alpha_1 y - ikz} \]
\[ \frac{iE_{y1} \alpha_1}{\mu_0 \mu_{r1} \omega} e^{-\alpha_1 y - ikz} \]

Magnetic fields
\[ H_2 = \left[ \frac{i}{\mu_0 \omega} \left( iE_{y2} k e^{\alpha_2 y - ikz} + E_{z2} \alpha_2 e^{\alpha_2 y - ikz} \right) \right] \]
\[ - \frac{iE_{z2} \alpha_2}{\mu_0 \omega} \left( \mu_{rd2} + \frac{\mu_{ro2}^2}{\mu_{rd2}} \right) \left( \mu_{rd2} + \frac{\mu_{ro2}^2}{\mu_{rd2}} \right) \left( \mu_{rd2} + \frac{\mu_{ro2}^2}{\mu_{rd2}} \right) \]
\[ - \frac{iE_{y2} \alpha_2}{\mu_0 \omega} \left( \mu_{rd2} + \frac{\mu_{ro2}^2}{\mu_{rd2}} \right) \left( \mu_{rd2} + \frac{\mu_{ro2}^2}{\mu_{rd2}} \right) \left( \mu_{rd2} + \frac{\mu_{ro2}^2}{\mu_{rd2}} \right) \]
Analysis 2

Boundary condition:

\[ \mathbf{a}_y \times (\mathbf{H}_1 - \mathbf{H}_2) = \sigma \mathbf{E}_T \]

\[
\begin{bmatrix}
\frac{1}{\mu_0 \mu_r \omega (\mu_{rd}^2 + \mu_{ro}^2)} \left( iE_{x1} \alpha_1 (\mu_{rd}^2 + \mu_{ro}^2) - E_{x1} \mu_0 \mu_r \omega \sigma (\mu_{rd}^2 + \mu_{ro}^2) + E_{x2} \mu_r (i\alpha_2 \mu_{rd} + \mu_{ro} k) \right) \\
0 \\
\frac{1}{\mu_0 \mu_r \omega} (E_{y1} k - E_{y2} \mu_r k + iE_{z1} \alpha_1 - E_{z1} \mu_0 \mu_r \omega \sigma + iE_{z2} \alpha_2 \mu_r) \\
\end{bmatrix}
= 0
\]

(1)

Normal components:

\[ \nabla \cdot \mathbf{E} = 0 \]

\[ E_{y1} = -\frac{iE_{z1}}{\alpha_1} k \quad E_{y2} = \frac{iE_{z2}}{\alpha_2} k \]

Substituting into (1)

\[
\begin{bmatrix}
0 & B_{12} \\
B_{21} & 0
\end{bmatrix}
\begin{bmatrix}
E_{x1} \\
E_{z1}
\end{bmatrix}
= 0
\]

\[ B_{12} = \frac{1}{\mu_0 \mu_r \omega (\mu_{rd}^2 + \mu_{ro}^2)} \left( i\alpha_1 (\mu_{rd}^2 + \mu_{ro}^2) - \mu_0 \mu_r \omega \sigma (\mu_{rd}^2 + \mu_{ro}^2) + \mu_r (i\alpha_2 \mu_{rd} + \mu_{ro} k) \right) \]

\[ B_{21} = \frac{i\alpha_1}{\mu_0 \mu_r \omega} + \frac{i\alpha_2}{\mu_0 \omega} - \sigma - \frac{ik^2}{\alpha_2 \mu_0 \omega} - \frac{ik^2}{\alpha_1 \mu_0 \mu_r \omega} \]
Analysis 3

\[
\begin{bmatrix}
0 & B_{12} \\
B_{21} & 0
\end{bmatrix}
\begin{bmatrix}
E_{x1} \\
E_{z1}
\end{bmatrix} = 0
\]

Eigenvalues & eigenvectors

\[B_{21}(k) = 0, \quad \begin{bmatrix} 0 \\ E_{z1} \end{bmatrix}\]

TM, reciprocal

\[B_{12}(k) = 0, \quad \begin{bmatrix} E_{x1} \\ 0 \end{bmatrix}\]

TE, nonreciprocal

Getting \(\alpha_1(k)\) and \(\alpha_2(k)\):
\[
\nabla \times (\bar{\mu}_r^{-1} \nabla \times \mathbf{E}) - \omega^2 \mu_0 \varepsilon_0 \varepsilon_r \mathbf{E} = 0, \quad \nabla \cdot \mathbf{E} = 0
\]

Region (1)

\[
\begin{bmatrix}
E_{x1}(-\alpha_1^2 - \varepsilon_0 \varepsilon_r \mu_0 \mu_1 \omega^2 + k^2) \\
iE_{z1} / \alpha_1 k(\alpha_1^2 + \varepsilon_0 \varepsilon_r \mu_0 \mu_1 \omega^2 - k^2) \\
E_{z1}(-\alpha_1^2 - \varepsilon_0 \varepsilon_r \mu_0 \mu_1 \omega^2 + k^2)
\end{bmatrix} = 0
\]

Region (2)

\[
\begin{bmatrix}
E_{x2} / \alpha_2 \mu_{rd2}^2 + \mu_{ro2}^2 (-\alpha_2^2 \mu_{rd2}^2 - \varepsilon_0 \varepsilon_r \mu_0 \mu_{rd2}^2 \omega^2 - \varepsilon_0 \varepsilon_r \mu_0 \mu_{ro2}^2 \omega^2 + \mu_{rd2} k^2) \\
iE_{x2} / \alpha_2 k(-\alpha_2^2 - \varepsilon_0 \varepsilon_r \mu_0 \omega^2 + k^2) \\
E_{x2}(-\alpha_2^2 - \varepsilon_0 \varepsilon_r \mu_0 \omega^2 + k^2)
\end{bmatrix} = 0
\]
Analysis 4

\[ \alpha_1 = \sqrt{-\varepsilon_0 \varepsilon_r \mu_0 \mu_{r1} \omega^2 + k^2} \]

\[ \alpha_2 = \sqrt{-\varepsilon_0 \varepsilon_r \mu_0 \mu_{r2} \omega^2 - \frac{\varepsilon_0 \varepsilon_r}{\mu_{rd2}} \mu_0 \mu_{ro2}^2 \omega^2 + k^2} \]

Dispersion equation:

\[
\left( -\mu_0 \mu_{r1} \omega \sigma (\mu_{rd2}^2 + \mu_{ro2}^2) + \mu_{r1} \right) \left( i \mu_{rd2} \sqrt{\frac{1}{\mu_{rd2}} \left( -\varepsilon_0 \varepsilon_r \mu_0 \mu_{ro2}^2 \omega^2 + \mu_{rd2} \left( -\varepsilon_0 \varepsilon_r \mu_0 \mu_{rd2} \omega^2 + k^2 \right) + \mu_{ro2} k \right)} + i (\mu_{rd2}^2 + \mu_{ro2}^2) \right) = 0
\]

indicating nonreciprocity
Results - Ferrite Substrate Permeability Tensor

ferrite parameters:

\( f_0 = 50 \, GHz \)
\( f_m = 10.6 \, GHz \)
\( \alpha = 0.05 \)

\( B_0 = 1.8 \, T \)
\( \mu_0 M_s = 0.38 \, T \)

Circularly polarized waves
Graphene Intraband and interband conductivities

graphene parameters:

\[ \mu_c = 0.1 \text{ eV} \]
\[ \tau = 0.2 \text{ ps} \]
\[ T = 300 \text{ K} \]
Dispersion Curves – Isolation Result

Phase nonreciprocity

Loss nonreciprocity

Substrate parameters:

\[ \varepsilon_{r1} = 1, \quad \mu_{r1} = 1 \]
\[ \varepsilon_{r2} = 9 \]

Isolation: -10 dB/cm
DC-Current-Based Graphene Plasmonic Isolator

- DC current
- Magnetic field
  - LHCP Forward
  - RHCP backward
- Ferrite
- DC magnetic field of the current above and below graphene
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❖ CONCLUSIONS & QUESTIONS
Broadside Coupled P-N Doped Graphene Strips

chemically doped

electrically doped
Magnetic Sensor

Port 1

Port 2

V₀

B₀

excited edge mode

output signal

graphene edge mode coupler

magnetic regions

non-magnetic regions

excited edge mode

no output signal

graphene edge mode coupler

non-magnetic regions

magnetic regions
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Phase shifter

Coupler

Isolators

Graphene-ferrite isolator

Magnetic sensor