Hybrid Staggered Perfectly Matched-layers in Non-Staggered Meshless Time-Domain Vector Potential Technique

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Abstract— In this paper a hybrid algorithm for the implementation of Perfectly Matched Layers (PMLs) in the meshless magnetic vector potential technique is proposed. Solving the wave equation in time-domain, the magnetic vector potential technique avoids using staggered node distributions which are needed for calculating the **E** and **H** fields when directly solving Maxwell's equations. However, implementing PMLs with stretched coordinate formulation requires auxiliary variables on a staggered (dual) node distribution. To avoid defining staggered nodes in the whole computational domain, a hybrid algorithm is proposed in this paper: The algorithm keeps a single set of nodes for the magnetic vector potential **A** inside the free space while it uses staggered nodes for **A** and auxiliary variables inside the PML. The hybrid algorithm is validated in a 2D rectangular waveguide and numerical reflection coefficients are compared for different thicknesses of the PML and for different orders of a polynomial conductivity profile inside the PML. A good agreement between theoretical results and converged solutions validates the approach.

Index Terms— Radial point interpolation, meshless methods, magnetic vector potential, wave equation, perfectly matched layer.

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Abstract

Solving the wave equation in time-domain, the magnetic vector potential technique avoids using staggered node distributions which are needed for calculating the $\mathbf{E}$ and $\mathbf{H}$ fields when directly solving Maxwell's equations. However, implementing PMLs with stretched coordinate formulation requires auxiliary variables on a staggered (dual) node distribution. To avoid defining staggered nodes in the whole computational domain, a hybrid algorithm is proposed.

Introduction: Meshless Methods

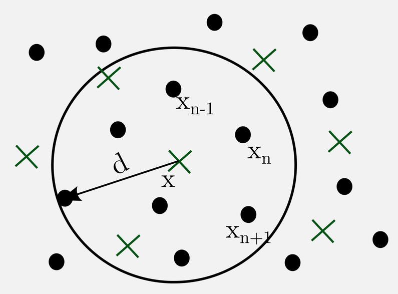
- No mesh generation
- Numerical solution of electromagnetic problems on cloud of points
- Accurate modelling of complex & multi-scale structures
- Remeshing → Adaptive node refinement

Radial Point Interpolation Method

In this method each field component at centre point of an influence domain is approximated by interpolation of surrounding nodes.

$$\langle u(\mathbf{x})\rangle = \Sigma_{n=1}^{N} a_n r_n(\mathbf{x}) + \Sigma_{m=1}^{M} b_m p_m(\mathbf{x})$$

$$\langle u(\mathbf{x})\rangle = [\Psi_1(\mathbf{x}), \Psi_2(\mathbf{x}), ..., \Psi_N(\mathbf{x})]\mathbf{U^e}$$



→ Spatial derivatives can be computed from basis functions derivatives

Magnetic Vector Potential Technique

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad \Rightarrow \quad \nabla^2 \mathbf{A} - \mu\epsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}$$

Staggered nodes for $\mathbf{E}$ & $\mathbf{H}$ — One set of nodes for $\mathbf{A}$

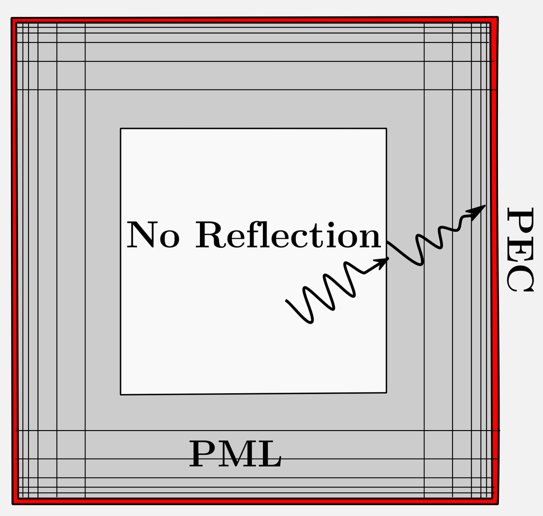
Perfectly Matched Layers

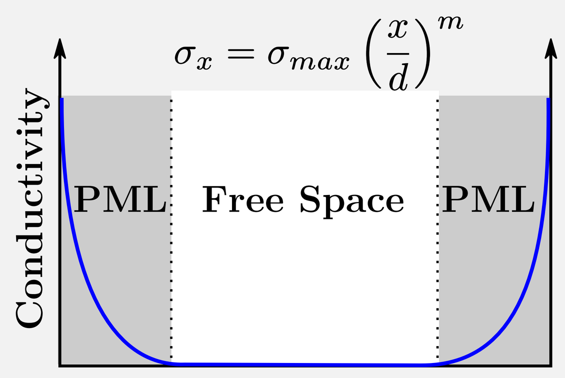
$$\frac{1}{s_x}\frac{\partial}{\partial x}\left(\frac{1}{s_x}\frac{\partial \mathbf{A}}{\partial x}\right) + \frac{1}{s_y}\frac{\partial}{\partial y}\left(\frac{1}{s_y}\frac{\partial \mathbf{A}}{\partial y}\right) - \mu\epsilon\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}$$

$$j\omega \mathbf{X}_1 = \frac{1}{s_x}\frac{\partial \mathbf{A}}{\partial x}$$

$$j\omega \mathbf{X}_2 = \frac{1}{s_x}\frac{\partial (j\omega \mathbf{X}_1)}{\partial x}$$

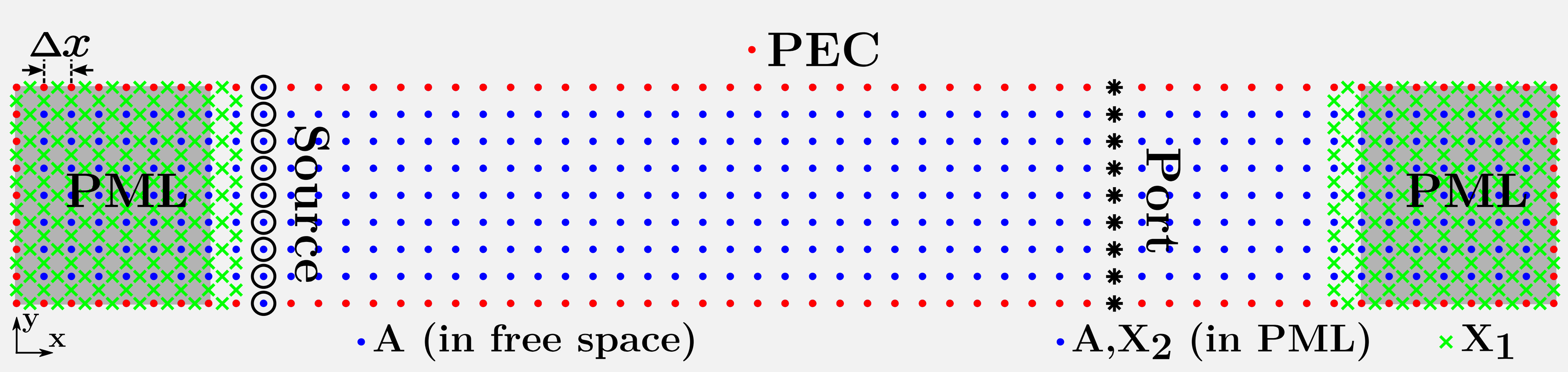
$$s_x = 1 + \sigma_x / j\omega\varepsilon$$



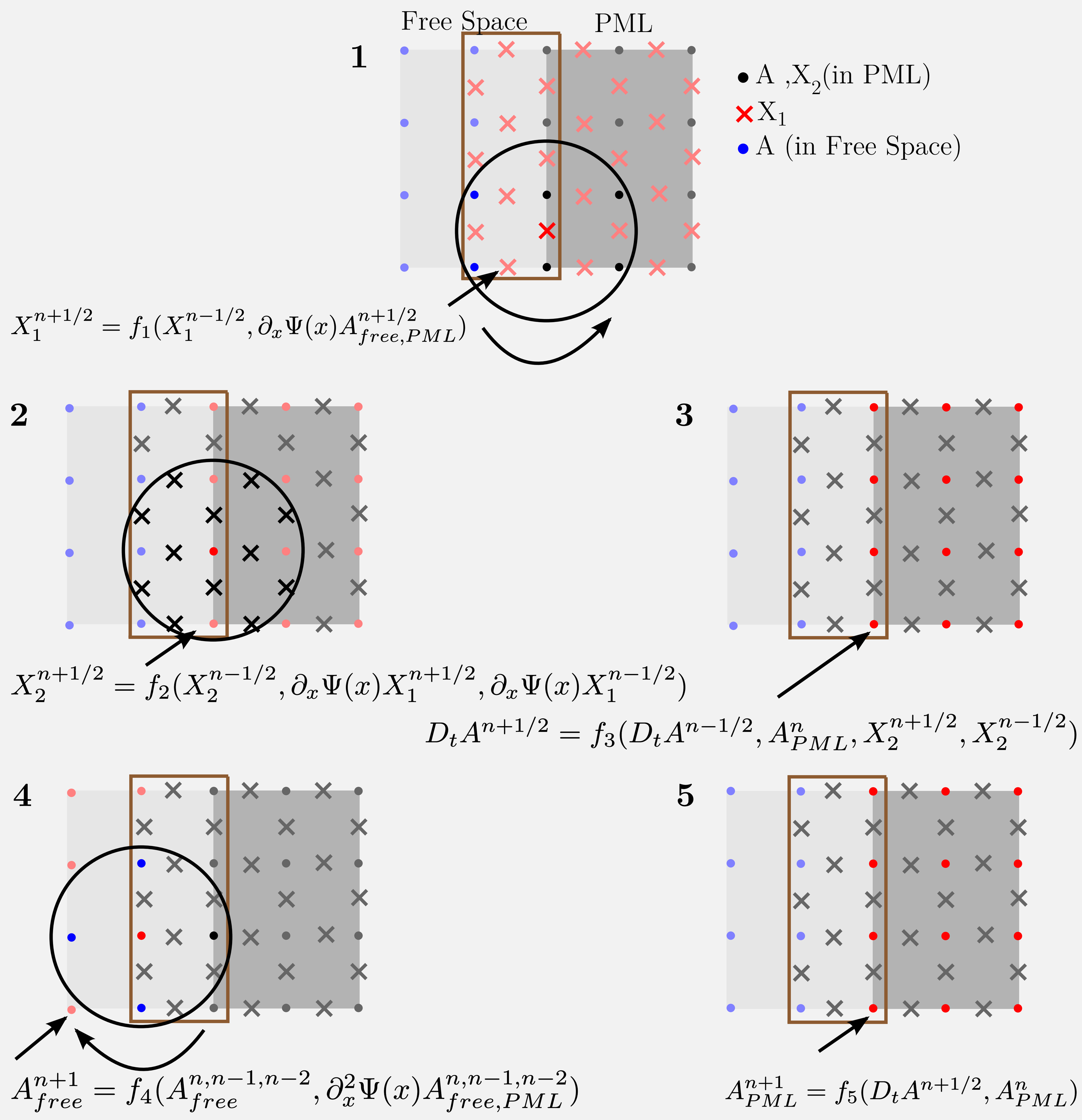


Hybrid Algorithm

- Free space: a single set of nodes for the $\mathbf{A}$.
- PMLs: staggered nodes for $\mathbf{A}$ and auxiliary variables $\mathbf{X}_1$ & $\mathbf{X}_2$.

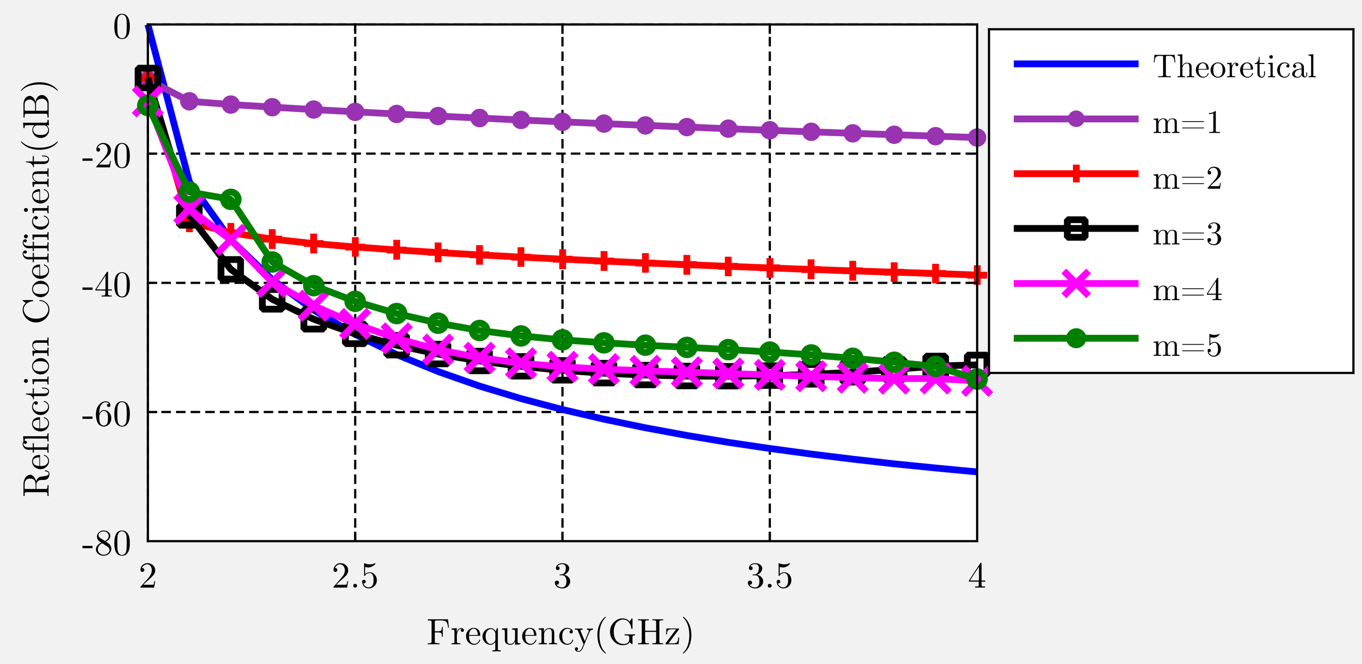


Updating Process

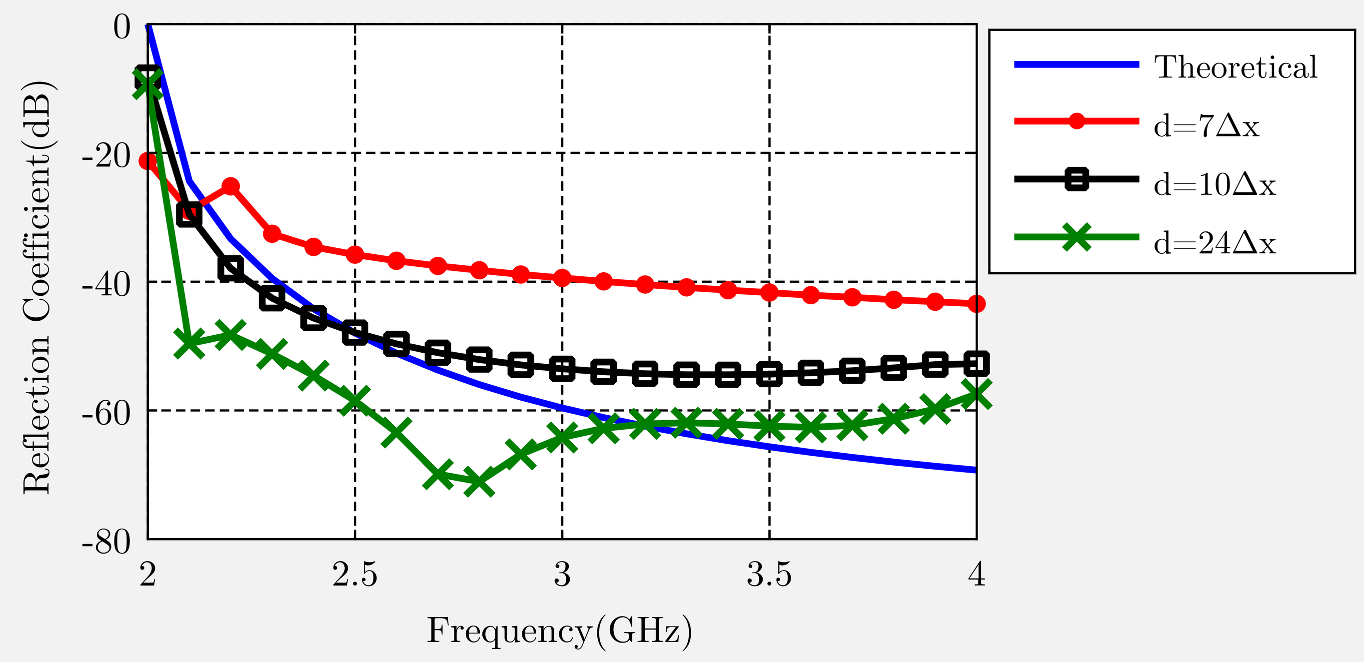


Numerical Results

- Different order of conductivity inside the PML when the thickness of PML is $d = 10\Delta x$.



- Different PML thicknesses when the order of conductivity profile is $m = 3$.



Conclusion

- A hybrid algorithm for the implementation of Perfectly Matched Layers (PMLs) in the meshless magnetic vector potential technique is proposed. This algorithm keeps a single set of nodes in free space while it uses staggered nodes in the PMLs.

Reference

[1] Y.S. Rickard, N.K. Georgieva, and W. Huang, "Application and optimization of PML ABC for the 3-D wave equation in the time domain," *IEEE Transactions on Antennas and Propagation*, vol. 51, no. 2, pp. 286–295, Feb. 2003.

Zahra Shaterian received her Bachelor and Master of Engineering degrees in Electrical and Electronic Engineering from Amirkabir University of Technology, Iran, in 2004 and K.N. Toosi University of Technology, Iran, in 2007, respectively. Currently, she is working toward her Ph.D. in Electrical and Electronic Engineering at the University of Adelaide, in the area of computational electromagnetics. Her research interests include computational electromagnetics, time-domain meshless methods, finite-difference time-domain analysis, and metamaterials and their applications in planar circuits.