Fast Solution of Multiple Slot-Coupled Waveguide Junctions Using the CBFM

Carl Toft[1], Rob Maaskant[1] and Johan Wettergren[2]
[1] Chalmers University of Technology, [2] RUAG Space AB

***Abstract*—A method for the fast analysis of a large number of waveguides coupled through rectangular apertures of arbitrary size and placement is presented. To enable the analysis of weakly excited slots, an accurate method-of-moments approach employing basis functions corresponding to a modal expansion of the electric field in the slot cavities is utilized, which includes both polarizations of the electric field at the slot apertures, as well as the transverse variation of the field across the slot. The characteristic basis function method (CBFM) reduces the memory requirements significantly and speeds up the computation, and is used to analyze a junction with two branch waveguides with very weakly excited slots, showing excellent agreement with the commercial FEKO software. The runtime of the program is considerably faster than FEKO for these geometries.**

***Index Terms*—Moment method, Microwave propagation, Waveguide slots, Aperture admittance, CBFM**

I. Introduction

DURING the initial design stage of slot array antennas, it is customary to perform a computationally intensive optimization in which a large number of slot positions and orientations are tried until a configuration that produces the desired radiation pattern and match has been achieved [1]. In this process, the radiating slots and the coupling slots that connect the waveguides are analyzed separately. If this optimization stage is to be completed within a reasonable time frame, software that is able to accurately and efficiently analyze waveguide sections coupled through slots is crucial. The relevant structure is shown in Fig. 1.

The aim of this paper is to present a very fast and memory-efficient method of moments (MoM) approach that can be used to accurately calculate the S-parameters of waveguide coupling junctions with rectangular, but otherwise completely arbitrary slots connecting the waveguides. The presented theory is a CBFM-enhanced version of the one developed in [2].

The moment method has been applied to waveguide problems extensively since Khac and Carson's pioneering work in the 70s [3], [4], where the moment method was used to solve an integral equation for the electric field at the slot aperture between the waveguides, derived by means of the Schelkunoff equivalence principle.

Since then the computing power available in modern computers has increased significantly, and the moment method approach for analyzing rectangular slots in rectangular waveguides has been refined and improved upon considerably. It has been used to predict resonant lengths of slots and to calculate the scattering parameters for a variety of waveguide structures, as well as many other applications [5]–[9].

However, in most approaches to this problem it has been assumed that the tangential electric field at the slot apertures has only a transverse component, i.e., there is no component that is directed longitudinally along the slot. It is also assumed that the field can vary longitudinally along the slot but is constant transversely across the slot. For moderately and strongly excited slots, it is a good assumption that corresponds well to measurements and full wave solvers. However, as shown by Petersen and Rengarajan, this assumption is not generally true and leads to incorrect results for weakly excited slots [10], [11]. For these, the longitudinal polarization of the electric field, as well as the transverse variation of the field across the slot, must be taken into account.

The contribution from these weakly excited slots is important in the analysis of one of the proposed scatterometer-radar antennas for the next generation of MetOp satellites. They will consist of a feed waveguide connected to a large number of branch waveguides, where the accumulated reflections from each slot must be taken into account for accurate results [1].

The developed theory includes these transverse effects by employing a set of entire domain trigonometric basis functions in the slot apertures that correspond to a modal expansion of the electric field in the slot cavities between the waveguides. Analytic expressions for all admittances and mode couplings for this set of basis functions have been derived, allowing for a very fast evaluation. This is achieved using a discrete modal expansion form for the waveguide Green's function which completely avoids complications arising from singularities in the self-terms.

In order to handle the large amount of basis functions that are needed to accurately model the aperture fields of many slot junctions, the characteristic basis function method (CBFM) [12] has been successfully employed to significantly reduce the memory requirements by reducing the number of basis functions per slot to at most four, making it possible to analyze geometries that would be intractable for a plain MoM code.

II. Analysis Method

By dividing the waveguide structure into a set of canonical regions, the requirement that the magnetic field be continuous across the boundaries between these regions gives rise to a set of integral equations for the electric fields at the slot apertures.

Consider the situation where a feed waveguide is connected to N branch waveguides. The slot connecting the feed waveguide to branch waveguide number q is shown in Fig. 2. By invoking the Schelkunoff equivalence principle [13], infinitely

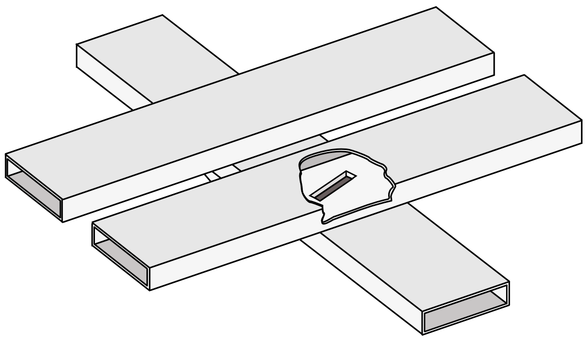

Fig. 1. Waveguide coupling junction with two branch waveguides. One branch waveguide has been cut open to show the slot connecting it to the feed waveguide below.

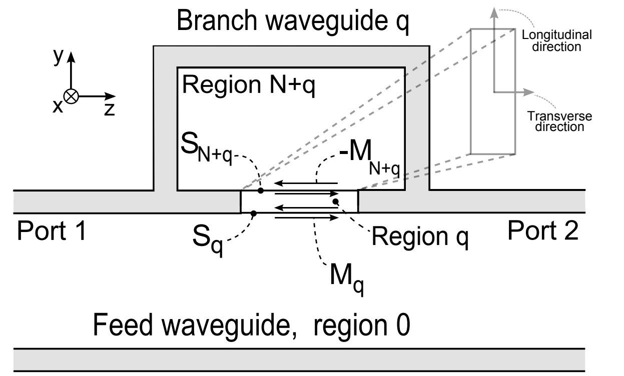

Fig. 2. Cross section showing feed waveguide and branch waveguide number q, together with a labeling of apertures, magnetic currents and canonical regions.

thin PEC sheets are added at each slot aperture S_k, and auxiliary surface magnetic currents are added directly above and below the sheets. These sheets decompose the geometry into a set of separate regions of two types: infinite rectangular waveguides and rectangular cavities, both of which are well-understood and have known analytical field solutions.

By letting $\overleftrightarrow{G}_X$ be the dyadic Green's function for the magnetic field generated by magnetic currents in region X, the continuity of the $\mathbf{H}$-field across the lower and upper apertures in cavity q is written as

$$\begin{aligned}\mathbf{H}^0_{\text{in}} + \sum_{k=1}^{N} \overleftrightarrow{G}_0|\mathbf{M}_k\rangle = -\overleftrightarrow{G}_q|\mathbf{M}_q\rangle + \overleftrightarrow{G}_q|\mathbf{M}_{N+q}\rangle, \text{ and} \\ -\overleftrightarrow{G}_q|\mathbf{M}_q\rangle + \overleftrightarrow{G}_q|\mathbf{M}_{N+q}\rangle = -\overleftrightarrow{G}_{N+q}|\mathbf{M}_{N+q}\rangle + \mathbf{H}^{N+q}_{\text{in}},\end{aligned} \tag{1}$$

respectively, where $q \in \{1, 2, \ldots, N\}$, and the shorthand notation

$$\overleftrightarrow{G}_X|\mathbf{M}_k\rangle = \iint_{S_k} \overleftrightarrow{G}_X(\mathbf{r}, \mathbf{r}_s)\mathbf{M}_k(\mathbf{r}_s)\mathrm{d}S \tag{2}$$

has been employed for the field generated by $\mathbf{M}_k$ in region X. The fields $\mathbf{H}^0_{\text{in}}$ and $\mathbf{H}^{N+q}_{\text{in}}$ are the fields incident in region 0 and $N+q$, respectively.

The expressions in Eq. (1) are a set of coupled integral equations for the unknown currents $\mathbf{M}_k$, and are solved using the method of moments, as follows.

Each unknown current $\mathbf{M}_k$ is expanded into a set of basis functions $\{\mathbf{m}_{kj}\}_{j=1}^{Q_k}$ according to

$$\mathbf{M}_k(\mathbf{r}) = \sum_{j=1}^{Q_k} V_{kj}\mathbf{m}_{kj}(\mathbf{r}), \tag{3}$$

where the coefficients V_{kj} are the new unknowns to be determined.

Forming the symmetric product of each integral equation in (1) with each basis function $\mathbf{m}_{kj}$ yields the system of algebraic equations

$$YV = I \tag{4}$$

for the vector V of expansion coefficients. The matrix Y is a matrix containing the self- and mutual-admittances between the basis functions in the apertures, both in the infinite waveguide regions and the regions formed by the slot cavities. Note that this approach takes into account the mutual coupling between all slots, and is thus valid also for closely spaced slots. The admittances take the form of the double surface integrals

$$\begin{aligned}&\langle \mathbf{m}_{kj}|\overleftrightarrow{G}_X|\mathbf{m}_{ni}\rangle = \\ &= \iint_{S_k} \mathrm{d}S\mathbf{m}_{kj}(\mathbf{r}) \cdot \iint_{S_n} \overleftrightarrow{G}_X(\mathbf{r}, \mathbf{r}_s)\mathbf{m}_{ni}(\mathbf{r}_s)\mathrm{d}S_s.\end{aligned} \tag{5}$$

I is a vector containing the coupling between the incident waveguide mode and the basis functions in the apertures.

Once the expansion coefficients have been solved for, the amplitudes of the propagating modes far into each port are calculated from the standard, modal expansion form of the infinite waveguide Green's function [2], [14].

The form of the admittances and mode couplings is dependent upon the choice of basis functions. A natural choice for them is to have trigonometric variation both along and across the slot in such a way that the expansion (3) corresponds to a modal expansion of the electric field inside the slot cavities. Each basis function will then also be polarized either completely along the slot, or completely across it. Other than the necessary truncation of the possibly infinite series expansion after a finite number of terms, no limiting assumption about the form of the aperture fields has been made.

Expressions for these basis functions, as well as the resulting admittances Y and mode couplings I can be found in [2], but are very unwieldy and are omitted here due to space constraints. [1] Seki's alternative expression for the waveguide Green's function was employed to facilitate the evaluation of the self admittances of the apertures in the infinite waveguide region [2], [15]. The Seki Green's function is written as two doubly infinite modal sums, one over y-modes and one over z-modes. To obtain accurate results, it is important to ensure that a sufficient number of waveguide modes have been included in the expansions. The z-travelling modes have to travel from the aperture to the walls of a "virtual cavity" and back, and since these decay exponentially with travelled distance, only around 15 need to be included. The y-travelling

[1] One property that is still worth noting is that a singularity-free modal spectrum for the Green's functions was used, leading to singularity-free kernels in (5) for the self-terms.

modes do not decay exponentially, and usually at least 5000 need to be included for convergence, as observed in [8].

The major drawback of this choice of basis functions is not the number of y-travelling modes needed to accurately evaluate the matrix elements, but rather that a large amount of basis functions may be needed to model the aperture electric field, and as the number of waveguides increases, the matrix Y in Eq. (4) quickly grows very large.

To overcome this, numerically-generated characteristic basis functions (CBFs) are used to reduce the matrix size. These basis functions are generated by first analyzing each branch waveguide in isolation. Accordingly, by exciting the truncated structure from each of the four ports, four aperture field distributions are obtained by solving this reduced problem, which are then taken as the macro domain basis functions and made linearly independent from one another by means of an SVD thresholding procedure. The number of basis functions is thus reduced to, at most, four per slot.

By denoting the CBFs in slot k by $\{\mathbf{M}_{kj}\}_{j=1}^{4}$, and the micro basis function by $\{\mathbf{m}_{kj}\}_{j=1}^{Q_k}$, the CBFs are expanded according to

$$\mathbf{M}_{kj} = \sum_{i=1}^{Q_k} V_{kj}^{i} \mathbf{m}_{ki}, \tag{6}$$

where the coefficients V_{kj}^{i} for the CBFs are determined as described above.

The admittances between CBFs are expressed in terms of the admittances (5) as [16]

$$\langle \mathbf{M}_{ki} | \overleftrightarrow{G}_X | \mathbf{M}_{lj} \rangle = \sum_{m=1}^{Q_k} V_{ki}^{m} \sum_{n=1}^{Q_l} \left[\langle \mathbf{m}_{km} | \overleftrightarrow{G}_X | \mathbf{m}_{ln} \rangle \right] V_{lj}^{n}. \tag{7}$$

The above equation represents a compression of the original admittance matrix that may contain the interaction between several hundred basis functions in each slot, to a very small admittance matrix of size, at most, $4N \times 4N$. It is thus possible to analyze a very large number (on the order of thousands) of branch waveguides before the size of the Y matrix becomes a problem.

Specifically, if n is the number of micro basis functions used in each aperture, the memory requirements for the two approaches scale as

$$\begin{aligned} M_{\text{plain}} &= 4N^2 n^2 \times \text{sizeof (complex number)} \\ M_{\text{CBFM}} &= 64N^2 \times \text{sizeof (complex number)}, \end{aligned} \tag{8}$$

where sizeof (complex number) is the memory required to store a complex number in the given architecture, often equal to 16 bytes.

Achieved compression factors $M_{\text{plain}}/M_{\text{CBFM}}$ are on the order of a few thousands, reducing the size of the moment matrix from megabytes or gigabytes to a few hundred kilobytes when CBFM is employed.

III. Numerical Results

The aim of the numerical experiments is threefold: (i) verify that the proposed algorithm produces accurate results by comparing with the commercial FEKO software, (ii) confirm that the transverse polarization and variation of the magnetic current must be included in order to obtain accurate results for weakly-excited slots, (iii) show that the CBFM-enhanced code agrees well with the plain MoM code, and compare the execution speed of both approaches.

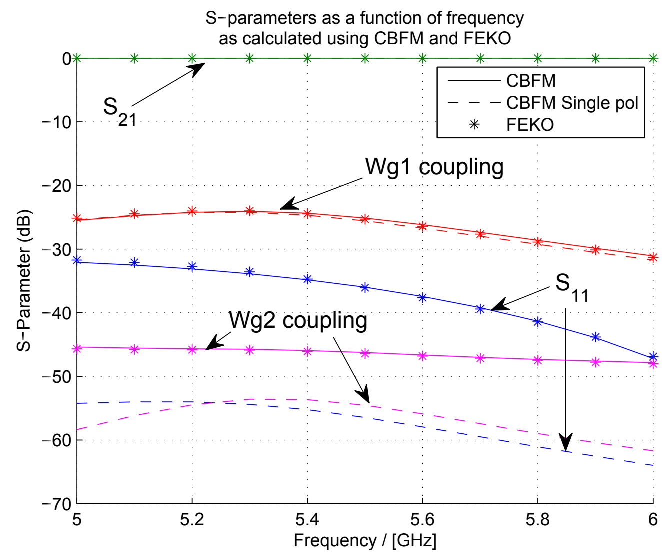

Fig. 3. S-parameters as a function of frequency for a waveguide junction consisting of a feed waveguide connected to two branch waveguides through weakly excited slots, as calculated by three different methods.

Fig. 3 shows the S-parameters as a function of frequency for a waveguide junction consisting of a feed waveguide with two branch waveguides placed above it, and accomplishes the first two goals. The slots are placed such that they are only weakly excited: the first is completely centered and the second offset $0.5\,\mathrm{mm}$ from the centerline. Both slots are directed longitudinally (no tilt). All waveguides have a rectangular cross-section of dimension $38.78\,\mathrm{mm} \times 10\,\mathrm{mm}$ and the slots are placed in the broad wall (i.e., the top or bottom wall) of the waveguides (see Fig. 1) with a longitudinal separation of $44.8\,\mathrm{mm}$ between the slot centers. The slot dimensions are $28\,\mathrm{mm} \times 3\,\mathrm{mm}$ and have a thickness of $1\,\mathrm{mm}$.

The figure shows the reflection, transmission and coupling into the branch waveguides as calculated using three different methods. The S-parameters were calculated both when including the transverse polarization and variation of the magnetic current (denoted "CBFM"), and also when neglecting these (denoted "CBFM Single pol"). Corresponding points as calculated by FEKO are included for comparison (denoted "FEKO"). FEKO meshes the entire waveguide structure and solves for the electric current in the waveguide walls; and hence, the transverse effects of the equivalent current is accounted for in these values.

The computations were performed in MATLAB R2014a on a computer using an Intel Core i5 CPU with a clock speed of 3.2 GHz. For the CBFM, the computation time was less than 3 seconds for each frequency. The corresponding calculation in FEKO took about one hour per frequency, though a direct comparison is not very meaningful since FEKO solves for the induced electric current in the walls of the entire structure.

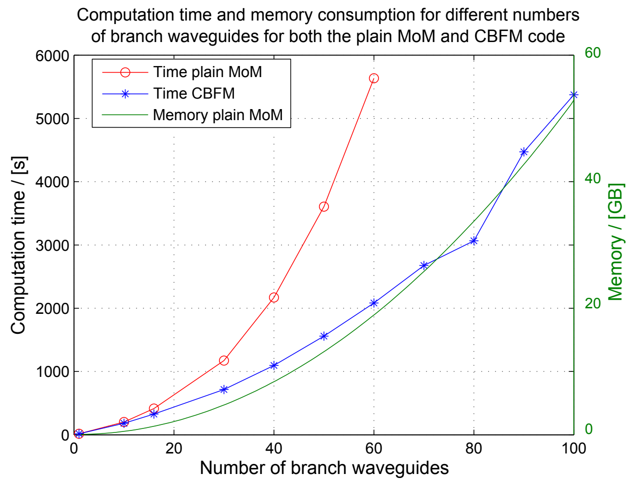

Fig. 4. Total time required by plain MoM and CBFM codes to calculate S-parameters at 5 GHz for waveguide junctions with different numbers of branch waveguides. The CBFM curve represents the worst case, i.e., no CBFs were reused in different slots. Memory requirement is only shown for plain MoM. According to Eqs. (8), the memory compression factor is 2500 in this case.

There is a very good agreement between FEKO (dots) and the values calculated by the developed program when the transverse effects were included (solid lines), demonstrating the excellent accuracy of the approach. Note also that neglecting the transverse effects (dashed lines) yields values for the reflection coefficient S_{11} and the power coupled into waveguide 2 that differ significantly from the values obtained through the other two methods, indicating that the transverse effects must be included in the analysis of weakly-excited slots if accurate values for these quantities are desired.

Each magnetic current was expanded into 200 basis functions, where half of them were used for the longitudinal polarization and the other half for the transverse polarization. In the modal expansion of the Green's function, 5000 y-propagating modes and 15 z-propagating modes were included and sufficient for convergence.

Lastly, the speed of the two approaches was compared by timing the execution speed of the plain MoM and CBFM codes when the number of branch waveguides was increased from one to 100 for the CBFM code, and from one to 60 for the plain MoM code. The results are shown in Fig. 4.

The two different codes require approximately the same amount of time for the small numbers of waveguides likely to be encountered in a practial situation, with CBFM being slightly faster. The CBFM code is faster since a much smaller system of equations is solved compared to the plain MoM code, and this size difference increases with increasing N. It should be stressed that the main advantage of using CBFM is the drastic reduction in memory usage compared to the plain MoM approach according to Eq. (8). In the present simulation, CBFM reduces the size of the moment matrix by a factor of 2500.

IV. Conclusion

A rigorous method for analyzing waveguides coupled through rectangular slots has been presented. This method is very efficient, allowing for a very fast analysis that is well-suited to be used as part of a larger optimization algorithm during the design of slot array antennas.

This method employs an expansion of the aperture electric field corresponding to a modal expansion of the electric field in the slot cavities and utilizes a discrete modal expansion of the waveguide Green's functions which avoids the problem of singularities when evaluating the self-terms. The electric field expansion includes the longitudinal polarization of the field, as well as the transverse field variation, and thus produces accurate results also for very weakly excited slots, as shown by comparison with FEKO.

Additionally, the CBFM was successfully implemented in order to significantly reduce the memory requirements. A procedure for generating the CBFs was presented and it was demonstrated that the CBFM code also runs faster than the plain MoM code.

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