Energy Patterns of the Prototype-Impulse Radiating Antenna (IRA)

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Abstract—A question that often comes up in the context of an IRA is “how is the transient energy from the pulser radiated in space”? Of course the electromagnetic fields (both $E$ and $H$), the power density and the energy density have their maximum on the bore sight. Power pattern is a well-defined frequency domain concept, but it is a cumbersome descriptor for hyperband antennas such as an IRA because of the multitude of frequencies involved. In this paper we explore the concept of an energy pattern which holds good both in time and frequency domains. An energy pattern is useful in visualizing where the transient energy provided to the IRA is going. It is further noted that the energy and power patterns are identical for a CW antenna, while they can be vastly different for pulsed antennas.

Index Terms—Impulse Radiating Antenna, Pulsed Antenna, Radiation Characteristics, Power Pattern, Energy Pattern

I. INTRODUCTION

An easy way to remember the performance parameters of an antenna is through an acronym “BRIDGE”. The individual letters denote:

- Beam width
- Radiation pattern
- Input impedance
- Directivity
- Gain
- Effective area

All of these parameters are well defined in the frequency domain, and are functions of frequency. In the context of pulsed antennas where many frequencies are simultaneously fed into the antenna, the use of frequency dependent parameters is useful, but cumbersome.

Let us consider a reflector type of an IRA as an example of a hyper band [1] antenna [2, 3, and 4]. The radiation pattern of an IRA is a strong function of frequency as reported in [5]. The lower frequencies of the input pulse have lower gain and large beam widths, while the higher frequencies have a higher gain and smaller beam widths. In the next section, we consider an energy pattern of the IRA which is a simple and unique descriptor. It is indicative of how the input energy is radiated into all of space.

II. ENERGY PATTERN

Let us denote the energy pattern of the IRA by $U(\theta, \phi)$ in the far field or Fraunhofer zone. This quantity is measured in Joules/steradians. Figure 1 shows the geometry and Cartesian and spherical sets of coordinates with origin at the center of the radiating aperture. The diameter and the focal length of the reflector are denoted by $D$ and $F$. The Fraunhofer zone is known to begin at an axial distance given by [6]

$$r_{far} = \frac{D^2}{2c\tau_{mr}}$$

where $c$ is the speed of light, $D =$ diameter of the reflector, and $\tau_{mr}$ is the maximum rate of rise of the voltage wave launched onto the reflector, which is not necessarily the rate of rise of the transient source waveform. There could be some risetime degradation in transporting the voltage pulse from the source to the wave launch.

Figure 1. The IRA geometry and coordinate system.

Let $\vec{E}_f(R, \theta, \phi, t)$ and $\vec{E}(R, \theta, \phi, \omega)$ denote the far electric field at an arbitrary location in time and frequency domains respectively. The energy pattern can be defined as

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in the far field, the electric field has a vanishing
and increases. This is $\phi = \pm 90$ degrees, $\phi = \pm 90$ degrees is zero due to
components. The
because the electric field is falling off like $r$. We can re-
energy pattern in the far field is independent of $r$, simply
electromagnetic fields depend on the radial distance $r$, but the
reference to Figure 1.

Parseval’s Theorem [7]. It is also observed while the
computes it in time domain or frequency domain, as per
that energy content of a signal is the same whether one
needs to be in the far field satisfying equation (1). It is noted
aperture. The relevant equations are given below [8], with
the magnetic current (tangential electric field) on the
arms. Initially the field on the aperture plane is computed.

The radiated field can then be computed by an integration of
far field can be estimated by the method of aperture
integration that takes into account the presence of the feed
planes. In fact, the radiation pattern from the
integration model will not give good results on the back- side
of the aperture plane. In summary, the aperture integration model gives good
responses in the forward direction, but not in the backward
direction. The numerical errors also increase as the
observation point approaches the plane of the aperture.

For this idealized aperture source, the radiated field in the
E-plane when $\phi = (\text{plus or minus})$ 90 degrees is zero due to
geometrical projection reasons. In the H plane, the radiated
field does not vanish for $\phi = (\text{plus or minus})$ 90 degrees,
because the equivalent source is radiating in free-space and this observation point is broadside to the magnetic current
source. If there were an infinite, perfectly conducting plate in
the plane of the aperture, there the field in the H-plane would
go to zero at $\pm$ 90 degrees due to the boundary condition of
$E_{\text{tan}} = 0$ on the screen. We also observe that the aperture
integration model will not give good results on the back-side
of the aperture plane. In fact, the radiation pattern from the
equivalent sources is the same in the back as in the front.

In summary, the aperture integration model gives good
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direction. The numerical errors also increase as the
observation point approaches the plane of the aperture.

III. ESTIMATION OF FAR FIELD BY APERTURE INTEGRATION METHOD

The electric field at any arbitrary observation point in the
far field can be estimated by the method of aperture
integration that takes into account the presence of the feed arms. Initially the field on the aperture plane is computed.

The magnetic current on the aperture is given by

and the electric field components in terms of the magnetic
current on the aperture are given by

From the estimation of all components of the electric field
in frequency domain, the total transient electric field is found
by Fourier inversion and then used in equation (3) to get the
energy pattern.

Some comments about the aperture integration method
outlined above are in order.

The aperture integration method for a hole in an infinite
screen is quite accurate in the direction of the main beam, but
the accuracy deteriorates as the angle $\phi$ increases. This is
due to the various approximations used to obtain an
analytical solution to this problem. In the EM model used
here for the dish, the tangential E and H-fields are normally
needed over the aperture, and are assumed to be zero
elsewhere on the aperture plane away from the dish. We use
only the tangential E-field distribution, however, and assume
the corresponding TEM value for the tangential H-field (and
this results in an equivalent magnetic current of $M = 2 E_a$
over the aperture).

For this idealized aperture source, the radiated field in the
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geometrical projection reasons. In the H plane, the radiated
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observation point approaches the plane of the aperture.

IV. ENERGY PATTERN OF THE PROTOTYPE IRA

In this section, we consider the prototype IRA and
evaluate its energy pattern. A photograph of this prototype
IRA is shown in Figure 2.
Figure 2. Photograph of the Prototype IRA.

The parameters of the prototype IRA are summarized in Table 1, where we have separately listed the geometry of the reflector and the pulser parameters. Having outlined the salient features of the prototype IRA, we now proceed to compute the radiated fields and the energy patterns.

The excitation voltage of the IRA is the total plate voltage situated at the focal point of the dish, and is denoted as $V_0(t)$. As outlined in [6], one possible representation of a fast-pulse is given by the expression

$$V(t) = V_0(1 + \Gamma) \left[ 0.5 \text{erfc} \left( -\frac{1}{\sqrt{2} \tau} \right) - 0.5 \text{erfc} \left( \frac{1}{\sqrt{2} \tau} \right) - u(t - t_0) \right]$$

(8)

TABLE 1. Antenna and pulser parameters of the prototype IRA

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Antenna Details</td>
<td></td>
</tr>
<tr>
<td>Reflector diameter D</td>
<td>3.66 m</td>
</tr>
<tr>
<td>Focal length F</td>
<td>1.22 m</td>
</tr>
<tr>
<td>F/D</td>
<td>0.33</td>
</tr>
<tr>
<td>Number of arms</td>
<td>4</td>
</tr>
<tr>
<td>Arm configuration</td>
<td>90 deg</td>
</tr>
<tr>
<td>Impedance $Z_{in}$</td>
<td>200 Ohms</td>
</tr>
<tr>
<td>Geometrical factor $f_g = Z_{in}/Z_0$</td>
<td>0.53</td>
</tr>
<tr>
<td>Polarization</td>
<td>Linear/Vertical</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b) Pulser Details</td>
<td></td>
</tr>
<tr>
<td>Peak voltage $V_p$</td>
<td>$\sim 60 \text{kV} \sim 120 \text{kV}$</td>
</tr>
<tr>
<td>Peak rate of rise $\frac{dV}{dt}_p$</td>
<td>$\sim 1.2 \times 10^{15} \text{V/s}$</td>
</tr>
<tr>
<td>Maximum rate of rise $t_{mr} = \frac{V_p}{\frac{dV}{dt}_p}$</td>
<td>$\sim 100 \text{ps}$</td>
</tr>
<tr>
<td>Pulse repetition frequency PRF</td>
<td>$\sim 200 \text{Hz}$</td>
</tr>
<tr>
<td>Pulse decay time $t_d$</td>
<td>$\sim 20 \text{ns}$</td>
</tr>
<tr>
<td>Duty cycle $\tau = t_d \text{PRF}$</td>
<td>$\sim 4 \times 10^6$</td>
</tr>
<tr>
<td>Peak power $P_{in} = \frac{V_p^2}{Z_{in}}$</td>
<td>$\sim 72 \text{MW}$</td>
</tr>
<tr>
<td>Average power</td>
<td>$\sim 72 \text{MW} \times 4 \times 10^6 \sim 288 \text{Watts}$</td>
</tr>
</tbody>
</table>
In equation (8), the term $\text{erfc}(.)$ denotes the complementary error function and $u(.)$ is the unit step (Heaviside) function. For the calculations here, the following parameters are used:

- $V_{\text{peak}} = 120$ (kV) (peak value of the transient waveform)
- $\Gamma = 0.006$ (amplitude adjustment factor)
- $\beta = 0.005$ (fall-time coefficient) = rise time / fall time = $100$ps / $20$ ns
- $\tau_r = 100$ (ps) (waveform rise time)
- $\tau_s = 1.0$ (ns) (time shift)

Figure 3 presents the waveform $V_o(t)$ for late and early times, respectively, and Figure 4 illustrates the time derivative $dV_o(t)/dt$ of the waveform. The spectral magnitude of the waveform is provided in Figure 5.

![Figure 3. Plot of the excitation waveform $V_o(t)$.](image)

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![Figure 4. Plot of the time rate of change of the excitation waveform $dV_o(t)/dt$.](image)

Figure 4. Plot of the time rate of change of the excitation waveform $dV_o(t)/dt$.

![Figure 5. Plot of the spectral magnitude of the excitation waveform.](image)

The calculation of the radiated far-field response of this antenna is done using an aperture integration method, which is described in [9]. The TEM field illuminating the dish is approximated and this forms the aperture field over the dish which is integrated to yield the impulse portion of the response. In addition, the radiation from the feed line currents and the source itself is determined and this provides the pre-pulse contribution to the field.

Overlay plots of the principal components of the transient E-fields for this antenna are presented in Figure 6 for the H-plane and in 7 for the E-plane.

As a check of these results, Giri [6] provides the following estimation of the peak value of the impulse portion of the normalized radiated E-field on axis $(\theta, \phi) = 0^\circ, 0^\circ$:

$$rE_{\text{peak}} \approx \frac{D}{4\pi c f_g} \left( \frac{dV}{dt} \right)_{\text{peak}} \sqrt{2} = \frac{D \sqrt{2} V_{\text{peak}}}{4\pi c f_g \tau_{\text{nr}}} \text{ (Volts)}$$

(9)

In this expression, $\frac{\partial V_o}{\partial t}|_{\text{peak}} \approx 1.184 \times 10^{15}$ (V/sec) from Figure 4 and $f_g = 1.06$. This provides an estimate of $rE_{\text{peak}} \approx 1.534$ MV, which is shown in the figures and agrees well with the present aperture integration calculations.

In these responses, it should be noted that the large impulse-like response occurs only in the boresight direction. It occurs in a very narrow beam around $(\theta, \phi) = 0^\circ, 0^\circ$, and even at a value of $\theta = 1^\circ$ there is a substantial reduction of the peak. Moreover, the waveforms become more dispersed in time as the angle $\theta$ increases. It is also noted that the waveforms in the H-plane and in the E-plane are different.

Although the $E_x$ and $E_z$ field components are not plotted here, it suffices to note that they are both either zero or negligible compared with the principal $E_y$ component.
Figure 6. Overlay of the principal \( (E_y) \) field components for various values of the angle \( \theta \), computed in the horizontal plane (the H-plane) in the far zone of the IRA. (for 120kV/100ps/20ns pulse excitation).

The spectral responses of the \( E_y \) field components in the H-plane and E-plane are also of interest. These are plotted as overlay plots for different values of the observation angle \( \theta \) in Figure 8 and Figure 9 respectively.

For the driving waveform of Figure 3 into an impedance of 200 Ohms (the net IRA impedance), the total energy delivered to the antenna can be estimated to be

\[
U_{\text{in}} = \frac{1}{200} \int_0^\infty V_e^2(t) dt \approx \frac{\Delta t}{200} \sum_{k=0}^{N-1} (V_{e_k}^2) = 0.727 \text{ Joules} \quad (10)
\]

Of interest is the spatial distribution density (in Joules/steradian) of the radiated energy from the IRA. Equation (3) is evaluated in both of the observation planes by and the results are presented in Figure 10 and Figure 11 for different scales for the y-axis.

Figure 7. Overlay of the principal \( (E_y) \) field components for various values of the angle \( \theta \), computed in the vertical plane (the E-plane) in the far zone of the IRA. (for 120kV/100ps/20ns pulse excitation).

Figure 8. Plot of the spectral magnitudes of the principal \( (E_y) \) field components computed in the horizontal plane (the H-plane) in the far zone of the IRA. (120kV/100ps/20ns pulse excitation.)
Figure 9. Plot of the spectral magnitudes of the principal \(E_y\) field components computed in the vertical plane (the E-plane) in the far zone of the IRA. (120kV/100ps/20ns pulse excitation.)

In this calculation, all vector components of the calculated E-field are used, even though \(x\) and \(y\)-components are very small. It is clear that the IRA sends more energy in the boresight direction. It is important to note that this radiated energy pattern will be different for other excitation sources, because it depends on the frequency content of the excitation voltage.

Figure 10. Plot of the radiated energy pattern in the horizontal (H) and vertical (E) planes for the IRA with the voltage excitation of Figure 3. (Log scale on the ordinate.)

V. CALCULATIONS FOR THE PROTOTYPE IRA WITH A DAMPED SINE WAVE EXCITATION

Because the radiated energy pattern depends on the waveform exciting the IRA, it is interesting to consider an alternate excitation function. In this section we examine a damped sine waveform with a center frequency of \(f_0 = 1\) GHz and exponential damping constant \(\alpha = 2 \times 10^8\) (1/sec). The peak amplitude of this waveform is adjusted to provide the same value of the previous pulse waveform, namely \(V_{max} = 120\) kV.

Figure 12 illustrates this waveform, and its derivative is shown in Figure 13. The spectral magnitude is provided in Figure 14. For this waveform, the resulting transient responses in the H- and E-planes are shown in Figure 15 and Figure 16, and the spectral responses are in Figure 17 and Figure 18. It is clear that the responses for this excitation are considerably different from those of the fast pulse.

Note that the estimated peak value of \(rE_{peak} = 1.009\) MV from equation (9) agrees well with the computed results. It is also worth noting that the low amplitude early time ringing in the waveforms of Figure 15 and Figure 16 is not a FFT problem, but rather, it is the pre-pulse response from the IRA.
Figure 12. Plot of the excitation waveform \( V_o(t) \).

Figure 13. Plot of the time rate of change of the excitation waveform \( dV_o(t)/dt \).

Figure 14. Plot of the spectral magnitude of the excitation waveform.

Figure 15. Illustration of the principal \((E_y)\) field component computed in the horizontal plane (the H-plane) in the far zone of the IRA. (120kV/1 GHz Damped sinusoidal pulse excitation.)

Figure 16. Illustration of the principal \((E_y)\) field component computed in the vertical plane (the E-plane) in the far zone of the IRA. (120 kV/1 GHz Damped sinusoidal pulse excitation.)
This is not the case for the damped sine wave, which has a rather narrow range of significant frequencies in the spectrum.

VI. SUMMARY

We have explored the concept of energy patterns (measured in Joules/steradian) for a pulsed antenna such as the IRA. In the case of CW antennas, the radiated power and energy patterns are the same. This is not the case for pulsed antennas. The radiated power pattern for a pulsed antenna is a strong function of frequency and can be computed for various frequencies.

In this paper, we have considered the prototype IRA and estimated its energy pattern for two different inputs with the same peak voltage amplitude of 120 kV. One input is a fast rising (100ps) - slowly decaying (20ns) mono-polar pulse, while the second is a 1 GHz damped sinusoidal voltage, which is bipolar.

The transient energies of the two input voltages have vastly different frequency components. The fast pulse has frequencies extending from DC to a few GHz, while the damped sinusoidal input is a moderate band source centered at 1 GHz. As a consequence of this, the energy patterns of the same prototype IRA are considerably different for the two input voltages.

In summary, the energy patterns are useful in visualizing where the transient energy provided to the IRA is being radiated in front of the antenna.

This paper is an adaptation of [10].

REFERENCES


