One can readily recognize the terms

$$P^{\rm diss} = (1-|\Gamma|^2) S^{\rm inc} A_{\rm e} \tag{4a}$$

$$P^{\rm re-radiated} = |\Gamma|^2 S^{\rm inc} A_{\rm e}, \tag{4b}$$

as the power dissipated in the antenna load and the re-radiated power, respectively. The last term in (3) should represent the net power which exits the antenna region under matched conditions, and which is denoted below as $P^{\rm exit}$.

III. The Poynting Theorem around a Receiving Antenna, Revisited

Consider a lossy scatterer illuminated by an incident plane wave $(\mathbf{E}^{\rm inc}(\mathbf{r}), \mathbf{H}^{\rm inc}(\mathbf{r}))$. Assume that the sources of the incident field are all located in the region $z<0$ away from the antenna. Once the antenna is introduced, the sources within its volume give rise to the scattered field $(\mathbf{E}^{\rm sc}(\mathbf{r}), \mathbf{H}^{\rm sc}(\mathbf{r}))$. The observable quantity around the antenna is then the total field $(\mathbf{E}^{\rm tot}(\mathbf{r}) = \mathbf{E}^{\rm inc}(\mathbf{r}) + \mathbf{E}^{\rm sc}(\mathbf{r}), \mathbf{H}^{\rm tot}(\mathbf{r}) = \mathbf{H}^{\rm inc}(\mathbf{r}) + \mathbf{H}^{\rm sc}(\mathbf{r}))$. Strictly speaking, the scattered field *everywhere* can only be obtained by mathematically subtracting the incident field from the total field, both of which can be observed independently.

Let us now apply the Poynting theorem in a volume V enclosed by closed surface A which contains all sources of $\mathbf{E}^{\rm sc}(\mathbf{r})$, while all the sources of $\mathbf{E}^{\rm inc}(\mathbf{r})$ are located outside the region in $z<-d$ (see Fig. 1):

$$\mathrm{Re}\,\frac{1}{2}\oiint_A \mathbf{E}^{\rm tot}\times\mathbf{H}^{\rm tot\star}\cdot d\mathbf{A} = -P^{\rm diss}, \tag{5}$$

where $d\mathbf{A}$ points outwards and $P^{\rm diss}$ is the power dissipated within V. If the lossy scatterer is embodied by a receiving antenna, then $P^{\rm diss}$ comprises the effects of the lossy load plus the ohmic losses in the antenna. The surface integral in (5) can be expanded as follows (see Stratton, [9, Sec. 9.26]):

$$\begin{aligned}\mathrm{Re}\,\frac{1}{2}\oiint_A &(\mathbf{E}^{\rm inc}+\mathbf{E}^{\rm sc})\times(\mathbf{H}^{\rm inc}+\mathbf{H}^{\rm sc})^\star\cdot d\mathbf{A}\\ &= \mathrm{Re}\,\frac{1}{2}\oiint_A \mathbf{E}^{\rm inc}\times\mathbf{H}^{\rm inc\star}\cdot d\mathbf{A}\\ &+\mathrm{Re}\,\frac{1}{2}\oiint_A\left(\mathbf{E}^{\rm inc}\times\mathbf{H}^{\rm sc\star}+\mathbf{E}^{\rm sc}\times\mathbf{H}^{\rm inc\star}\right)\cdot d\mathbf{A}\\ &+\mathrm{Re}\,\frac{1}{2}\oiint_A \mathbf{E}^{\rm sc}\times\mathbf{H}^{\rm sc\star}\cdot d\mathbf{A} = -P^{\rm diss}.\end{aligned} \tag{6}$$

In view of the fact that $(\mathbf{E}^{\rm inc}(\mathbf{r}), \mathbf{H}^{\rm inc}(\mathbf{r}))$ have no sources inside the region V, we note that

$$\mathrm{Re}\oiint_A \mathbf{S}^{\rm inc}\cdot d\mathbf{A} = 0. \tag{7}$$

or, by $A = A_{\rm illum}\cup A_{\rm shadow}$,

$$P^{\rm through} - P^{\rm inc} = 0, \tag{8}$$

where $P^{\rm inc}$ has been defined in (2) and

$$P^{\rm through} = \iint_{A_{\rm shadow}} \mathbf{S}^{\rm inc}\cdot d\mathbf{A}. \tag{9}$$

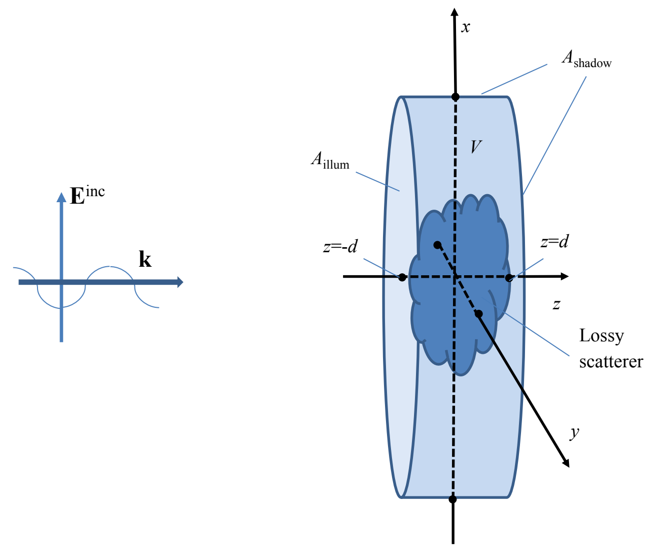


Fig. 1. A plane wave is incident upon a lossy scatterer embedded in slab of volume V that is enclosed by the closed surface surface $A = A_{\rm illum}\cup A_{\rm shadow}$, where $A_{\rm illum}$ and $A_{\rm shadow}$ are the illuminated and shadowed areas, respectively.

The cross term in (6) is sometimes referred to as the "extracted power":

$$-P^{\rm extr} = \mathrm{Re}\,\frac{1}{2}\oiint_A\left(\mathbf{E}^{\rm inc}\times\mathbf{H}^{\rm sc\star}+\mathbf{E}^{\rm sc}\times\mathbf{H}^{\rm inc\star}\right)\cdot d\mathbf{A} \tag{10}$$

This term expresses the interaction between the incident and scattered fields. The portion of the integral (10) over $A_{\rm illum}$, where the fields propagate in opposite directions, is zero (see Appendix I). Therefore,[1]

$$-P^{\rm extr} = \mathrm{Re}\,\frac{1}{2}\iint_{A_{\rm shadow}}\left(\mathbf{E}^{\rm inc}\times\mathbf{H}^{\rm sc\star}+\mathbf{E}^{\rm sc}\times\mathbf{H}^{\rm inc\star}\right)\cdot d\mathbf{A}. \tag{11}$$

The last term in (6) is

$$P^{\rm sc} = \mathrm{Re}\,\frac{1}{2}\oiint_A \mathbf{E}^{\rm sc}\times\mathbf{H}^{\rm sc\star}\cdot d\mathbf{A}, \tag{12}$$

which has been named "scattered power" [10]–[12]. However, it is not the net exiting power, since it is always accompanied by $P^{\rm extr}$, as seen in the following form of the power balance:

$$P^{\rm inc} = P^{\rm diss} + (P^{\rm through} + P^{\rm sc} - P^{\rm extr}). \tag{13}$$

The quantity inside the parentheses in (13) is the balance between the power which enters the volume V and the power which is dissipated. Therefore, it can be interpreted as the net power $P^{\rm exit}$ which exits V via A, and is given by

$$P^{\rm exit} = P^{\rm inc} + P^{\rm sc} - P^{\rm extr}, \tag{14}$$

in which we have substituted $P^{\rm through} = P^{\rm inc}$ (see (8)).

[1] The spectral version of (11) is the same as the Optical theorem, see Appendix I.

If we omit of $P^{\rm extr}$ from (13) and name $P^{\rm sc}$ the "scattered power," then it may lead to definitions with little physical meaning. For example, Pozar [11, Eq. 9] points out that a definition of "receiving efficiency" as the ratio of absorbed power to the sum of the absorbed power and $P^{\rm sc}$ alone, may lead to absurdities, such as attaining maximal efficiency when the load is totally mismatched. Pozar indeed notes that the notion of aperture efficiency "incorporates the physically appealing idea that maximizing the absorbed power for a given incident field should maximize receiving aperture efficiency." However, contrary to the standard definition [2] of the aperture efficiency used herein, the definition in [11, Eq. 10] makes use of A_e rather than A (see (2)). The definition in [11, Eq. 10] is in agreement with [2] only for large apertures with uniform distribution.

Using (14), Eq. (13) can now be written as

$$P^{\rm inc} = P^{\rm diss} + P^{\rm exit}. \tag{15}$$

The powers $P^{\rm exit}$ and $P^{\rm inc}$ can be observed in two corresponding scenarios, with and without the scatterer, respectively. However, there is no way to split their balance into $P^{\rm sc}$ and $P^{\rm extr}$ based on power measurements only.

A. Example: PEC plate

If the scatterer is a large thin PEC plate which coincides with the $z = 0$ plane, then the induced current on it will be close to the Physical Optics (PO) solution $\mathbf{J}_s = -2\widehat{\mathbf{z}}\times\mathbf{H}^{\rm inc}$. In this case, the $P^{\rm sc} = 2P^{\rm inc}$ and also $P^{\rm extr} = 2P^{\rm inc}$. Therefore,

$$P^{\rm exit} = P^{\rm inc}. \tag{16}$$

This is simply a statement that the plate reflects back all the energy that is incident upon it. It can be seen that the physical scattered power is identical to the reflected power, which can defined as $P^{\rm sc}_{\rm illum}$, where

$$P^{\rm sc}_{\rm illum} = {\rm Re}\,\frac{1}{2}\iint\limits_{A_{\rm illum}} \mathbf{E}^{\rm sc}\times\mathbf{H}^{\rm sc\star}\cdot d\mathbf{A}, \tag{17}$$

such that

$$P^{\rm sc} = P^{\rm sc}_{\rm illum} + P^{\rm sc}_{\rm shadow}, \tag{18}$$

where $P^{\rm sc}_{\rm shadow}$ is defined over the shadowed surface in a similar manner. In this example, since $\mathbf{E}^{\rm sc}$ is symmetric about $z = 0$, we have $P^{\rm sc}_{\rm shadow} = P^{\rm sc}_{\rm illum}$. If $P^{\rm sc}_{\rm shadow}$ is interpreted as an actual power, this result would indicate that the plate has scattered *twice the incident power*, and that the scatterer has been able to extract *twice the incident power* from the incident wave. This would require the existence of an effective aperture which is *twice* as large as $A_{\rm shadow}$. Such a conjecture is without foundation in practice.

In summary, the definition of the physical scattered power for this example obeys

$$P^{\rm exit} = P^{\rm inc} = P^{\rm sc}_{\rm illum}. \tag{19}$$

B. Relationship between aperture efficiency and $P^{\rm exit}$

Let us now consider the case of a perfectly matched antenna and recall (3), now with $\Gamma = 0$. Next, let us compare this equation side by side with (15):

$$P^{\rm inc} = \eta_{\rm a}\iint\limits_{A_{\rm illum}} \mathbf{S}^{\rm inc}\cdot d\mathbf{A} + (1-\eta_{\rm a})\iint\limits_{A_{\rm illum}} \mathbf{S}^{\rm inc}\cdot d\mathbf{A} \tag{20a}$$

$$P^{\rm inc} = P^{\rm diss} + P^{\rm exit}. \tag{20b}$$

Since it is obvious that

$$P^{\rm diss} = \eta_{\rm a}\iint\limits_{A_{\rm illum}} \mathbf{S}^{\rm inc}\cdot d\mathbf{A}, \tag{21}$$

it follows that

$$P^{\rm exit} = (1-\eta_{\rm a})\iint\limits_{A_{\rm illum}} \mathbf{S}^{\rm inc}\cdot d\mathbf{A}, \tag{22}$$

as mentioned after (4). Eq. (22) states that $P^{\rm exit}$ can attain any value between 0 and $P^{\rm inc}$, depending on the value of η_a. An interesting case is that of $\eta_a = 1$. While hard to attain in practice, one can approach this value by letting the aperture distribution on transmit to be as uniform as possible e.g., by realizing an array with uniform aperture distribution. In this case, the power received by a normally incident plane wave of matching polarization will be delivered to the load in its entirety. Therefore, an antenna of this type is equivalent to a perfect absorber. Lower aperture efficiency would lead to non-zero values for $P^{\rm exit}$.

Example: arrays of $\lambda/2$ dipoles: A planar receiving an array of dipoles, which is backed by a ground plane and is uniformly excited has been analyzed by Pozar [13, Sec. II-C]. The expression for maximum absorbed power, derived therein, is indeed equal to the incident power. Note, however, that this result is applicable for any aperture with $\eta_a = 1$. A similar array without a ground plane has also been analyzed in [13, Sec. II-A], showing that 50% of the power is absorbed and the other 50% exits the antenna region. By our definition, this array has an aperture efficiency of $\eta_a = 0.5$ due to the fact that the array scatters symmetrically into both the illuminated and shadowed regions. Again, this result is applicable to any aperture with $\eta_a = 0.5$, and in particular to any aperture which scatters symmetrically in two opposite directions.

IV. Defining the Physical Aperture and Aperture Efficiency for Arbitrary Antennas

The expression (14) for $P^{\rm exit}$ is dependent on the specification of $A_{\rm illum}$, and as such is not unique. The larger the $A_{\rm illum}$, the larger is the value of $P^{\rm inc}$; hence $P^{\rm exit}$ becomes larger with an increase in $A_{\rm illum}$. In the former examples, it was implicitly assumed that the illuminated area coincides with the physical size of the antenna hardware. This can serve as an approximate definition of the physical area for aperture antennas, for which the IEEE standard [2] is applicable.

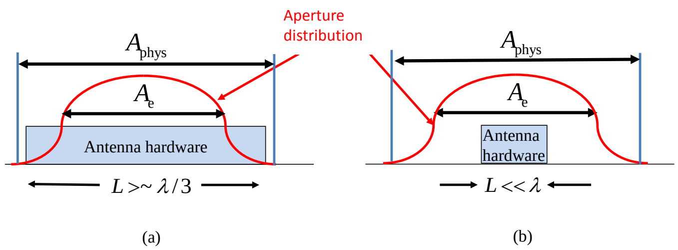


Fig. 2. Possible definitions of the physical aperture A_{phys}: (a) for a resonant length and above, (b) for a small antenna. L is the size of the antenna hardware. $A_e = \eta_a A_{\text{phys}}$ is the effective aperture.

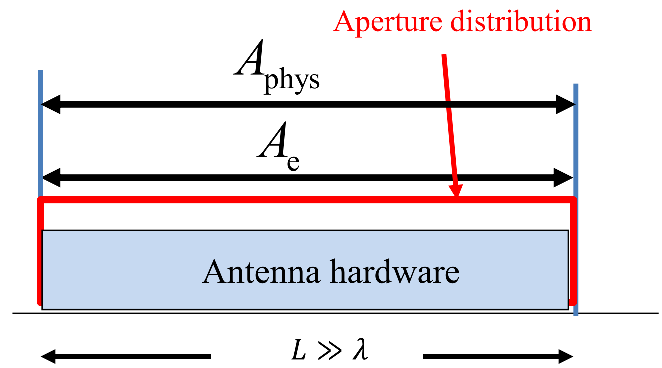


Fig. 3. A large aperture with perfectly rectangular pulse type aperture distribution has $\eta_{\text{a}} = 1$.

A. The physical aperture

If we wish to evaluate P^{exit} for arbitrary antennas, we would need a unique value of A_{illum}. This would be the Physical Aperture A_{phys} of the antenna. To this end, we record the distribution of the scattered field over a planar aperture which is close to the antenna in the shadow region. This distribution is expected to drop off gradually outside the antenna hardware, since the field cannot fall abruptly to zero at the edge of the hardware. This effect adds a small area to the the size of the hardware. While this effect can be negligible for large apertures, it becomes dominant for small antennas. A_{phys} would be close to the size of antenna hardware for large reflectors, horns or arrays; however it is more subjective when it comes to, say, wire antennas, (see Fig. 2). This definition relies upon the extent of equivalent sources rather than on the actual electric current sources alone.

Suppose now that the aperture distribution is large and uniform. Then, the physical aperture can be well defined, and $A_{\text{illum}} = A_{\text{e}}$, i.e., $\eta_{\text{a}} = 1$ (see Fig. 3). Then,

$$P^{\text{diss}} = (1 - |\Gamma|^2)P^{\text{inc}} = (1 - |\Gamma|^2)S^{\text{inc}}A_e. \tag{23}$$

However, as noted above, A_{illum} can be arbitrarily defined as larger than the one depicted in Fig. 3, say by choosing $A_{\text{illum},1} = \eta_{\text{a}}^{-1}A_{\text{e}}$. Then, the incident power is $P_1^{\text{inc}} = \eta_{\text{a}}^{-1}P^{\text{inc}}$ and

$$P^{\text{diss}} = (1 - |\Gamma|^2)\eta_{\text{a}}P_1^{\text{inc}} = (1 - |\Gamma|^2)S^{\text{inc}}A_e, \tag{24}$$

such that P^{diss} is unaffected by the choice of A_{illum}.

B. Example: the $\lambda/2$ dipole

In this example, we attempt to assess the physical aperture size of the $\lambda/2$ dipole shown in Fig. 4. The Poynting vector is sampled over a plane in the shadowed region, 0.017λ behind the dipole. The Poynting vector distributions along H- and E-lines over this plane, as depicted in Fig. 4, are shown in Figs. 5(a) and 5(b), respectively. The H- and E-line distributions are about 0.33λ and 0.66λ long, respectively, hence the physical aperture A_{phys} is approximately $0.22\lambda^2$. Since the effective aperture, based on directivity, is $A_e = 0.13\lambda^2$, the aperture efficiency of the dipole appears to be $\eta_a \simeq 0.59$. This value may be incorrectly interpreted as representing a general rule whereby one half of the received power has to be scattered. See the following section for further comment.

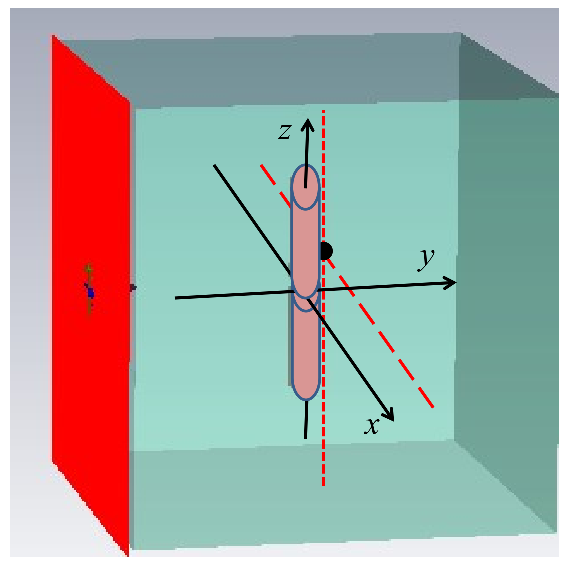
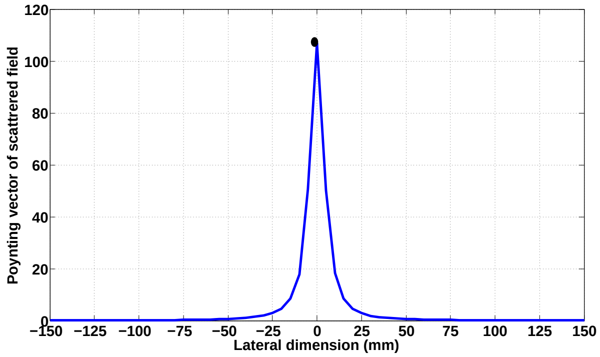


Fig. 4. A $\lambda/2$ dipole, aligned along the z-axis, is subject to illumination by a Z-polarized plane wave traveling in the $+y$ direction in a CST® simulation. The embedding box is $1\lambda \times 1\lambda \times 1\lambda$ large. S^{sc} is recorded along the H-line ($y = 0.017\lambda$, $z = 0.13\lambda$) and E-line ($y = 0.017\lambda$, $x = 0$), shown as dashed and dotted lines, see Fig. 5. The two lines intersect at the black dot.

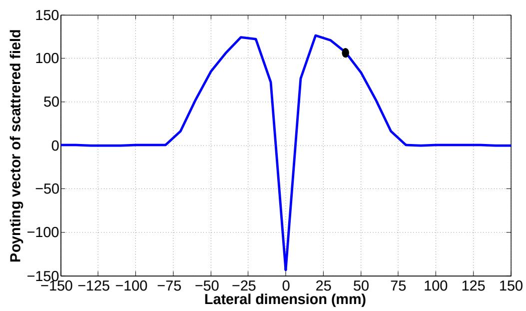
V. Conclusion

A straightforward application of the Poynting theorem on a surface which encloses a receiving antenna resulted in the observation that the power that exits the interior volume V via A can only be measured as a combination of extracted and scattered powers, and is related to the arbitrary specification of the illuminated aperture of the antenna. A unique definition for the physical area was subsequently provided. This definition is independent of the antenna type and can be used for wire antennas as well. Hence, the definition of the aperture efficiency applies to an arbitrary antenna. It is seen that when the aperture efficiency tends to unity,and the antenna is perfectly matched, then all the received power within the physical aperture as defined above is converted into absorbed power, i.e., the antenna acts as a perfect absorber. There is no “scattered power” in this case.

The above conclusions leave open the interpretation of the



(a) $S^{\rm sc}$ along the H-line (dashed line in Fig. 4).



(b) $S^{\rm sc}$ along the E-line (dotted line in Fig. 4).

Fig. 5. The Poynting vector $S^{\rm sc}$ for the dipole in Fig. 4, plotted vs. the H- and E-lines, defined in therein. 97.4% of $\iint P^{\rm sc} dA$ reside within a $\lambda/3$ band around the dipole in the H-plane and 0.6λ in the E-plane, making the physical aperture about $A_{\rm phys} \simeq 0.22\lambda^2$ large. Since the effective aperture, based on directivity, is $A_e = 0.13\lambda^2$, the aperture efficiency of the dipole appears to be about $\eta_a \simeq 0.59$.

power that is equivalently dissipated in the antenna impedance. This power, equal to the power delivered to the load under perfect match conditions, has been interpreted as "scattered power," leading to rather confusing conclusions [1],[3]–[8], [10], including statements such as "When the antenna system is conjugately matched i.e., $Z_L = Z_A$, only half of the available power from the equivalent source goes to the matched load while the other half goes to the antennas internal resistance (the real part of Z_A) and therefore, must be reradiated back to the space. Thus, the antenna efficiency can be no more than 50%, which is the paradox" that has been identified in [12]. It has been suggested in [14] that this power can be found in the internal impedance of the transmitter.

Appendix I
Spectral Power Balance

A. Definitions of Spectral Fields

Consider the case where a plane wave propagating in the $+z$-direction is incident upon a receiving antenna in Fig. 1. Take $A = A_{\rm illum} \cup A_{\rm shadow} \cup A_{\rm grazed}$, where $A_{\rm illum}$ and $A_{\rm shadow}$ are the illuminated and shadowed areas at $z = \mp d$, respectively. $A_{\rm illum}$ and $A_{\rm shadow}$ are assumed to be sufficiently large relatively to the size of the scatterer such that most of the energy of $E^{\rm sc}(z = \mp d)$ over $z = \mp d$ is contained within these areas.

Let us define the plane wave spectral expansion the tangential components of $(\mathbf{E}^{\rm sc}, \mathbf{H}^{\rm sc})$ over $A_{\rm illum}$ and $A_{\rm shadow}$ as follows:

$$\widetilde{\mathbf{E}}^{\rm sc}_{\rm T}(\mathbf{k}_t; \mp d) = \iint\limits_{z=\mp d} \mathbf{E}^{\rm sc}_{\rm T}(x, y, \mp d) e^{\jmath \mathbf{k}_t \cdot \mathbf{r}} d^2 r \tag{25a}$$

$$\begin{aligned}\widetilde{\mathbf{H}}^{\rm sc}_{\rm T}(\mathbf{k}_t; \mp d) &= \iint\limits_{z=\mp d} \mathbf{H}^{\rm sc}_{\rm T}(x, y, \mp d) e^{\jmath \mathbf{k}_t \cdot \mathbf{r}} d^2 r \\ &= \mp \frac{\widehat{\mathbf{z}}}{Z_{\rm modal}(\mathbf{k}_t)} \times \widetilde{\mathbf{E}}^{\rm sc}_{\rm T}(\mathbf{k}_t; \mp d),\end{aligned} \tag{25b}$$

where $\mathbf{k}_t = k_x \widehat{\mathbf{x}} + k_y \widehat{\mathbf{y}}$ and $Z_{\rm modal}(\mathbf{k}_t)$ is the appropriate modal impedance. The incident field is

$$\begin{pmatrix} \mathbf{E}^{\rm inc} \\ \mathbf{H}^{\rm inc} \end{pmatrix} = \begin{pmatrix} \widehat{\mathbf{x}} \\ \frac{1}{\eta}\widehat{\mathbf{y}} \end{pmatrix} E_0 e^{-\jmath k z}, \quad \eta = \sqrt{\mu/\epsilon} \tag{26}$$

and its spectral representation over the planes $z = \mp d$ is

$$\begin{pmatrix} \widetilde{\mathbf{E}}^{\rm inc}(\mathbf{k}_t, z = \mp d) \\ \widetilde{\mathbf{H}}^{\rm inc}(\mathbf{k}_t, z = \mp d) \end{pmatrix} = \begin{pmatrix} \widehat{\mathbf{x}} \\ \frac{1}{\eta}\widehat{\mathbf{y}} \end{pmatrix} (2\pi)^2 E_0 e^{\jmath \pm k d} \delta(\mathbf{k}_t). \tag{27}$$

B. Spectral integrations for power

We first show that the contribution to $P^{\rm extr}$ from the illuminated surface is annulled due to the absence of interaction between the incident and scattered fields. The illuminated portion of (11) can be re-cast in spectral terms using Parseval's theorem:

$$\begin{aligned} & - P^{\rm extr}_{\rm illum} \\ =& {\rm Re} \frac{1}{2} \iint\limits_{z=-d} \left(\mathbf{E}^{\rm inc} \times \mathbf{H}^{\rm sc\star} + \mathbf{E}^{\rm sc} \times \mathbf{H}^{\rm inc\star} \right) \cdot (-\widehat{\mathbf{z}})\, dx dy \\ =& -{\rm Re} \frac{1}{8\pi^2} \iint \left(\widehat{\mathbf{x}} E_0 e^{\jmath k d} 4\pi^2 \delta(\mathbf{k}_t) \times \left[-\widehat{\mathbf{z}} \times \frac{\widetilde{\mathbf{E}}^{\rm sc\star}_{\rm T}(\mathbf{k}_t; -d)}{Z^\star_{\rm modal}(\mathbf{k}_t)} \right] \right. \\ & \left. - \widetilde{\mathbf{E}}^{\rm sc}_{\rm T} \times \frac{1}{\eta} \widehat{\mathbf{y}} E_0^\star e^{-\jmath k d} 4\pi^2 \delta(\mathbf{k}_t) \right) \cdot \widehat{\mathbf{z}}\, d^2 k_t \\ =& -{\rm Re} \frac{1}{2} \left(E_0 e^{\jmath k d} \frac{\widetilde{E}^{\rm sc\star}_x(0,0;-d)}{Z^\star_{\rm modal}(0,0)} - \widetilde{E}^{\rm sc}_x(0,0;-d) \frac{1}{\eta} E_0^\star e^{-\jmath k d} \right). \end{aligned} \tag{28}$$

Now $Z^\star_{\rm modal}(0,0) = Z_{\rm modal}(0,0) = \eta$. The minus sign between the first term and its complex conjugate in (30) makes the expression in parentheses purely imaginary, therefore we have

$$P^{\rm extr}_{\rm ileum} = 0. \tag{29}$$

The spectral expression for $P^{\rm extr}$ is then evaluated over the shadowed surface only:

$$\begin{aligned}
&-P^{\rm extr}\\
=&\mathrm{Re}\,\frac{1}{2}\iint_{z=d}\left(\mathbf{E}^{\rm inc}\times\mathbf{H}^{\rm sc\star}+\mathbf{E}^{\rm sc}\times\mathbf{H}^{\rm inc\star}\right)\cdot\widehat{\mathbf{z}}\;dxdy\\
=&\mathrm{Re}\,\frac{1}{8\pi^2}\iint\left(\widehat{\mathbf{x}}E_0e^{-\jmath kd}4\pi^2\delta(\mathbf{k}_t)\times\left[\widehat{\mathbf{z}}\times\frac{\widetilde{\mathbf{E}}^{\rm sc\star}_{\rm T}(\mathbf{k}_t;d)}{Z^\star_{\rm modal}(\mathbf{k}_t)}\right]\right.\\
&\left.+\widetilde{\mathbf{E}}^{\rm sc}_{\rm T}\times\frac{1}{\eta}\widehat{\mathbf{y}}E_0^\star e^{\jmath kd}4\pi^2\delta(\mathbf{k}_t)\right)\cdot\widehat{\mathbf{z}}\;d^2k_t\\
=&\mathrm{Re}\,\frac{1}{2}\left(E_0e^{-\jmath kd}\frac{\widetilde{E}^{\rm sc\star}_x(0,0;d)}{Z^\star_{\rm modal}(0,0)}+\widetilde{E}^{\rm sc}_x(0,0;d)\frac{1}{\eta}E_0^\star e^{\jmath kd}\right).\\
=&\mathrm{Re}\,\frac{1}{2\eta}\left(E_0e^{-\jmath kd}\widetilde{E}^{\rm sc\star}_x(0,0;d)+\widetilde{E}^{\rm sc}_x(0,0;d)E_0^\star e^{\jmath kd}\right),
\end{aligned}\tag{30}$$

since $Z_{\rm modal}(0,0)=\eta$.

Similarly, the expression for $P^{\rm sc}_{\rm shadow}$ (the shadow portion of the integral in (6)) can be cast as

$$\begin{aligned}
P^{\rm sc}_{\rm shadow}&=\mathrm{Re}\,\frac{1}{8\pi^2}\iint\widetilde{\mathbf{E}}^{\rm sc}_{\rm T}(\mathbf{k}_t;d)\times\left[\widehat{\mathbf{z}}\times\frac{\widetilde{\mathbf{E}}^{\rm sc\star}_{\rm T}(\mathbf{k}_t;d)}{Z^\star_{\rm modal}(\mathbf{k}_t)}\right]\cdot\widehat{\mathbf{z}}\;d^2k_t\\
&=\mathrm{Re}\,\frac{1}{8\pi^2}\iint\widetilde{\mathbf{E}}^{\rm sc}_{\rm T}(\mathbf{k}_t;d)\cdot\frac{\widetilde{\mathbf{E}}^{\rm sc\star}_{\rm T}(\mathbf{k}_t;d)}{Z^\star_{\rm modal}(\mathbf{k}_t)}d^2k_t.
\end{aligned}\tag{31}$$

Substitute all terms into the expression for $P_{\rm shadow}$ and rearrange as follows:

$$\begin{aligned}
P_{\rm shadow}&=P^{\rm through}-P^{\rm extr}+P^{\rm sc}_{\rm shadow}\\
&=\mathrm{Re}\,\frac{1}{2\eta}E_0e^{-\jmath kd}\left(E_0e^{-\jmath kd}+\widetilde{E}^{\rm sc}_x(0,0;d)\right)^\star\\
&\qquad+\mathrm{Re}\,\frac{1}{2\eta}\left(\widetilde{E}^{\rm sc}_x(0,0;d)E_0^\star e^{\jmath kd}\right.\\
&\qquad\left.+\frac{\eta}{4\pi^2}\iint\frac{\widetilde{\mathbf{E}}^{\rm sc}_{\rm T}(\mathbf{k}_t;d)\cdot\widetilde{\mathbf{E}}^{\rm sc\star}_{\rm T}(\mathbf{k}_t;d)}{Z^\star_{\rm modal}(\mathbf{k}_t)}d^2k_t\right).
\end{aligned}\tag{32}$$

C. The Optical Theorem

The central term in (32) is

$$P^{\rm extr}=-\mathrm{Re}\,\frac{1}{\eta}E_0e^{-\jmath kd}\widetilde{E}^{\rm sc\star}_x(0,0;d).\tag{33}$$

Eq. (33) can be seen as a form of the Optical Theorem [15]. It can be interpreted to mean that the contribution to $P^{\rm extr}$ comes only from the spectral component $k_x=k_y=0$, for which interaction occurs between $\mathbf{E}^{\rm inc}$ and $\mathbf{E}^{\rm sc}$.

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