Fast Data Acquisition and Real-Time Processing Techniques for Through-the-Wall Radar Imaging

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Abstract—One of the main challenges in through-the-wall radar imaging (TWRI) is the long data acquisition and/or data processing time. The movement of the target during the long data collection time will cause smearing and target displacement in the through-the-wall image. Rapid data processing is critical to the real-time monitoring of targets behind the wall. In this paper, we present three techniques for fast data acquisition and real-time processing for TWRI. The first technique is based on compressive sensing incorporating the layered media Green’s function and is capable of dealing with targets behind multilayered building walls. Next, we formulate the beamforming algorithm for through-the-wall imaging with multiple-input multiple-output (MIMO) radar which is real-time in data collection. Finally, we propose a real-time processing algorithm for radar imaging through single or multilayered walls. Numerical results are presented to show the efficiency and the promising applications of the above techniques for fast data acquisition and real-time processing in TWRI.

Index Terms—TWRI, fast data acquisition, real-time processing, compressive sensing, MIMO radar, diffraction tomography.

I. INTRODUCTION

The high-resolution and noninvasive imaging of targets through visually opaque obstacles, such as walls and doors has sparked a growing interest in through-the-wall radar imaging (TWRI) in both military and commercial applications, such as homeland security, urban counter-terrorism, search and rescue missions [1]-[7].

Through-the-wall radar images the targets behind the wall by transmitting ultrawideband (UWB) electromagnetic (EM) waves and processing the reflected signal from the wall and the targets. In Synthetic Aperture Radar (SAR) and Ground Penetrating Radar (GPR), the targets are usually stationary. However, in through-the-wall imaging the targets may move when the radar is collecting the scattered signal. During a long data collection time, the target may have moved such that the beamformed solution is no longer valid as the target is not in its previous position anymore. This will cause smearing, displacement and false targets in the image. Fast data acquisition avoids image smearing and imaged target displacements and allows prompt actionable intelligence in TWRI. In Sections II and III, we present a generalized compressive sensing approach for reduced data collection and fast data acquisition in TWRI, and an efficient beamforming algorithm for through-the-wall imaging with multiple-input multiple-output (MIMO) radar which is real-time in data acquisition. Then, we present the detailed formulation and some representative results of a generalized 2-D diffraction tomographic algorithm for real-time TWRI. In the following, we present some backgrounds on the above three topics.

It is well known that achieving high imaging resolution in both cross-range and down-range requires the synthesis of a long aperture and transmission of an UWB signal. These requirements result in a large amount of space-time/space-frequency data and long data acquisition time in TWRI. Reduction of the data volume and acceleration of the data acquisition is important in TWRI, as it accelerates processing, and subsequently, allows prompt actionable intelligence [8]-[10]. It also relaxes constraints on system aperture and bandwidth, and creates different design and deployment paradigms, which are more flexible than those underlying conventional SAR operations. The capability of compressive sensing (CS) to reconstruct a sparse signal from far fewer non-adaptive measurements provides a new perspective to address these challenges in TWRI. In CS, it has been shown that a signal/image, which is sparse or has a sparse representation in some basis, can be captured from a small number of random non-adaptive linear projections onto the measurement basis. The original signal can be reconstructed with incomplete measurement data through convex optimization that uses the sparsity as an important a
priori information [11]-[13]. There have been several approaches that apply CS to radar imaging [14]-[18]. In [13], [14], [17], CS is applied to synthetic aperture radar to achieve the high resolution imaging with far fewer measurement. Gurbuz et al. proposed a novel CS data acquisition and imaging approach for step frequency and impulse ground penetrating radars in [15, 16]. In [8], [18] similar data acquisition and target reconstruction strategies were applied for stepped frequency continuous wave radar through-wall imaging in the framework of CS. In [9], CS is applied to impulse-through-the-wall radar imaging. Other recent works on application of CS to TWRI are those reported in [19]-[20].

Most of the existing literatures on TWRI with CS deal with only a single layer wall, in which the ray tracing technique is used to calculate the wave propagation time as it travels through the wall. In many urban sensing applications, we often encounter situations to detect and identify targets inside a building with multilayered inner walls or walls separated by a hallway [3]-[5], [7]. This is challenging and is beyond the capability of existing through-the-wall beamformers using ray tracing techniques. In Section II of this paper, we expand on application of the CS to the problem of imaging of targets located behind either a single or multilayered building wall through the incorporation of a generalized forward model in the imaging algorithm to take into account and compensate for the wall effects.

Existing TWRI are mainly discussed within the framework of monostatic SAR. One of the main drawbacks of monostatic radar system is the long data acquisition time to synthesize the aperture. It takes about a few minutes and over an hour to synthesize 1-D linear and 2-D rectangular apertures in the laboratory measurement, respectively [7], [22]. In Section III, we address the problem of through-the-wall imaging with multiple-input multiple-output (MIMO) radar. The MIMO concept using multiple antennas at both the transmitter and receiver was first proposed to improve communication performance and later extended to radar systems and has been attracted increasing interest in TWRI in the recent years [25]-[29]. The MIMO radar system does not need to synthesize an aperture thus is real-time in data acquisition. Moreover, the spatially distributed multiple transmitters and receivers in MIMO radar increase the diversity of target illumination and scattering by viewing the target from multiple aspects [30].

In addition to fast data acquisition, the fast processing of the received signal to form an image of the targets behind the walls is critical to real-time monitoring of targets in TWRI. Up to present, several effective TWRI algorithms that take into account the wave reflection, bending, and delay effects due to the presence of the wall have been proposed, such as delay-and-sum (DS) beamforming algorithm, linear inverse scattering algorithms and time reversal based imaging algorithms, etc [6], [31]-[33]. Although successful imaging results can be obtained by using these algorithms, they are based on the pixel-by-pixel reconstruction of the image. The computation time increases significantly with the increasing of the number of pixels of the image, making these algorithms still not applicable to real-time processing, particularly when one deals with large scale imaging scenarios or 3-D imaging. TWRI algorithms must be computationally efficient, so that the imaging of the targets can be achieved within a few seconds/minutes for 2-D/3-D imaging with a portable computer. In [2], [34] 2-D and 3-D diffraction tomographic (DT) algorithms are presented for TWRI with a single layer homogeneous wall. The DT algorithm requires much less computational resources and is much more computationally efficient due to the easy implementation of the algorithm with Fast Fourier Transform and inverse Fast Fourier Transform (FFT/IFFT). In Section IV of this paper, we present a generalized diffraction tomographic algorithm for 2-D real-time TWRI that is capable of imaging of targets behind single or multilayered building walls.

II. COMPRESSIVE SENSING FOR TWRI

A. Forward model for imaging through layered walls

Figure 1 shows a typical scenario of through-the-wall imaging with monostatic SAR. The radar antenna moves along a scan line parallel to the homogeneous wall from \(L_{\text{min}}\) to \(L_{\text{max}}\). The location of the \(m\)-th transceiver is denoted as \(r_m\). The radar operating frequency ranges from \(f_{\text{min}}\) to \(f_{\text{max}}\) with frequency step \(\Delta f\). The wall region may consist of single or multilayered walls.

Under the point target model, the received scattered field can be written as

\[
E_s(r_m, k_n) = \int_{D_{r_m}} d\mathbf{r} \sigma(\mathbf{r}) G(r_m, r, k_n) G(\mathbf{r}, r_m, k_n)
\]

(1)

where \(E_s(r_m, k_n)\) is the received scattered field at the \(m\)-th antenna location, \(G(r_m, r, k_n)\) is the background layered media Green’s function, \(\sigma(\mathbf{r})\) is the reflectivity of the target, \(k_n\) is the wave number of the \(n\)-th operating frequency.

![Fig.1 Measurement Configuration of TWRI](image)

Given the received signal at all antenna locations, the imaging function is given by [35]

\[
I(\mathbf{r}) = \int_{k_{\text{min}}}^{k_{\text{max}}} \int_{k_{\text{min}}}^{k_{\text{max}}} d\mathbf{r_m} dk_n E_s(r_m, k_n) G^{-2}(r_m, \mathbf{r}, k_n)
\]

(2)

where \(k_{\text{min}}\) and \(k_{\text{max}}\) are the wave-numbers of the minimum and maximum working frequency. It is noticed that in the
Above equation the reciprocity principle \( G(\mathbf{r}_{im}, \mathbf{r}, k_n) = G(\mathbf{r}, \mathbf{r}_{im}, k_n) \) has been used. To compute the image in (2), an efficient evaluation of the layered media Green’s function is required. This is complicated and time consuming as it generally involves the numerical or semi-analytical evaluation of the Sommerfeld integrals [36]. In order to compromise between the computation time and the exact evaluation of the Green’s function, we assume that the target is located in the far field of the antenna and approximate the far field Green’s function as [6], [7]

\[
G(\mathbf{r}_{im}, \mathbf{r}, k_n) = T \frac{e^{jk_n \mathbf{R}_{im}}}{4\pi R_{im}} \tag{3}
\]

where \( R_{im} \) is the distances from the \( m \)-th antenna location to the target and \( T \) is the transmission coefficients from the radar antenna to the target and will be given in Section II B.

Given the reflectivity of the target, by substituting (3) into (2) and ignoring the real-valued coefficient \( 4\pi R_{im} \), the received signal at the \( m \)-th receiver can be written as

\[
E_i(\mathbf{r}_{im}, k_n) = \int d\mathbf{r} \sigma(\mathbf{r}) e^{jk_n R_{im}} T(\mathbf{r}, \mathbf{r}_{im}, k_n) R_{im}^{-2} \tag{4}
\]

and the image can be then reconstructed as

\[
I(\mathbf{r}) = \int_{R_{im}}^{R_{max}} \int_{R_{imin}}^{R_{max}} d\mathbf{r}_{im} dk_n E_i(\mathbf{r}_{im}, k_n) e^{jk_n R_{im}} \cdot T^{-2}(\mathbf{r}, \mathbf{r}_{im}, k_n) R_{im}^2 \tag{5}
\]

### B. Transmission through Multilayered Walls

![Fig.2 Transmission through Multilayered Media](image)

Fig.2 Transmission through Multilayered Media

By incorporating the far field approximation of the layered media Green’s function, the imaging algorithm can be easily generalized to the imaging of targets located behind multilayered homogeneous walls, such as a brick wall, poured concrete wall or a hollow concrete wall. Let us consider the multilayered homogeneous media shown in Figure 2, where the permeability, permittivity and thickness of the \( n \)-th region are denoted as \( \mu_n \), \( \epsilon_n \) and \( d_n \), respectively. Applying the boundary condition at the interface of each layer, the generalized reflection coefficient can be derived, as detailed in [23]

\[
\tilde{R}_{i+1,i} = \frac{R_{i+1} + \tilde{R}_{i+1,i+2} e^{j2k_{i+1}z(d_{i+1} - d_i)}}{1 + R_{i+1} \tilde{R}_{i+1,i+2} e^{j2k_{i+1}z(d_{i+1} - d_i)}} \tag{6}
\]

where \( k_z \) is the normal component of wavenumber \( k \) in the \( i \)-th layer, \( \tilde{R}_{i+1} \) is the generalized reflection coefficient from the \( i+1 \)-th layer to the \( i \)-th layer, and \( R_{i+1,j} \) is the reflection coefficient from the \( i+1 \)-th layer to the \( j \)-th layer. The local half-space reflection and transmission coefficients are given as

\[
R_j = \frac{k_{iz} - k_{jz}}{k_{iz} + k_{jz}}, \quad T_j = \frac{2k_{iz}}{k_{iz} + k_{jz}} \tag{7}
\]

The generalized transmission coefficient from the first layer to the \( N \)-th layer can also be derived from the continuous boundary condition at the interface of each layer [23]

\[
T_{1N} = \prod_{p=1}^{N-1} \exp(-jk_{iz}d_{Np-1} + jk_{iz}d_1) \cdot \frac{1 - R_{p-1} \exp(j2k_{iz}(d_p - d_{p-1}))}{1 - R_p \exp(j2k_{iz}(d_p - d_{p-1} - d_p))} \tag{8}
\]

For a single layer homogeneous wall, the total reflection and transmission coefficients in (8) and (9) reduces to [6]

\[
\tilde{R}_{12} = \frac{1 - \exp(j2k_{iz}(d_2 - d_1))}{1 - \exp(j2k_{iz}(d_2 - d_1))} \tag{10}
\]

\[
T_{13} = \frac{1 - \exp(j2k_{iz}(d_2 - d_1))}{1 - \exp(j2k_{iz}(d_2 - d_1))} \tag{11}
\]

### C. Imaging through multilayered building walls with CS

Referring to Figure 1, we assume there are \( M \) receiving antenna locations and a transmitted stepped-frequency signal of \( N \) narrowband waveforms. The received signal at the \( m \)-th antenna with the \( n \)-th frequency bin is given by (4). Through discretization, (4) can be written in the following matrix form

\[
\mathbf{z}_m = \mathbf{\Psi}_m \mathbf{s} \tag{12}
\]

where \( \mathbf{z}_m \) is the measured data at the \( m \)-th antenna location with the \( i \)-th element \([\mathbf{z}_m]_i = E_i(\mathbf{r}_{im}, k_n)\), \( \mathbf{s} \) is a weighted indicator vector defining the target space, which is a \( K \times L \) pixel image and vectorized into an \( K \times L \) column vector. The target space and the measured data is related to \( \mathbf{\Psi}_m \) whose \((n, p)\)\( \text{th} \) element is given by

\[
[\mathbf{\Psi}_m]_{n,p} = T^2(\mathbf{r}_{ip}, \mathbf{r}_{im}, k_n) e^{j2k_n R_{im}} R_{im}^{-2} \tag{13}
\]

In contrast to conventional back-projection beamforming, which uses data measured at all antenna locations and all frequencies, in the CS through-the-wall imaging approach, we utilize a set of random measurements at a random subset of \( J \) frequencies at each antenna location, \( J \leq N \). Then the measurement data at the \( m \)-th antenna can be written as

\[
\mathbf{y}_m = \mathbf{\Phi}_m \mathbf{\Psi}_m \mathbf{s} \tag{14}
\]
where $\Phi_m$ is a $J \times N$ measurement matrix in which each row has only one nonzero element, which is equal to one. The location of this nonzero element corresponds to the index of the measured frequency bin in the transmitting frequency sequence. In essence, the above measurement matrix is formed by randomly choosing $J$ rows from $\Psi_m$.

For coherent processing of the received data, we superpose the matrix equation in (14) for all $M$ illumination locations to form a large matrix

$$
y = \Phi \Psi s = \Theta s$$

(15)

where

$$
y = \begin{bmatrix} y_1^T, y_2^T, \cdots, y_M^T \end{bmatrix}^T, \quad \Phi = \text{diag} \left[ \Phi_1, \Phi_2, \cdots, \Phi_M \right],$$

$$
\Psi = \begin{bmatrix} \Psi_1^T, \Psi_2^T, \cdots, \Psi_M^T \end{bmatrix}^T, \quad \Theta = \Phi \Psi
$$

If the target space is sparse, which is generally valid for most TWRI applications, the reconstruction of $s$ in the above equation can be achieved by solving the following sparse constraint optimization problem

$$
\hat{s} = \arg\min \| \tilde{s} \| \quad \text{s.t.} \quad y = \Theta s
$$

(16)

For radar imaging problem, the measurement data are inevitably contaminated by noise. The data at the $m$-th antenna location can then be written in the following form,

$$
y_m = \Theta_m s + u_m
$$

(17)

where $u_m$ is the measurement noise at the $m$-th antenna location. For noisy measurements, (15) can be rewritten as

$$
y = \Theta s + u
$$

(18)

where $u = \begin{bmatrix} u_1^T, u_2^T, \cdots, u_M^T \end{bmatrix}^T$.

By separating the real and imaginary part of both sides of (15), it can be written in the following equivalent form

$$
\tilde{y} = \Gamma \tilde{s}
$$

(19)

where

$$
\tilde{y} = \begin{bmatrix} \text{Re}(y^r) & \text{Im}(y^r) \end{bmatrix}^T, \quad \tilde{s} = \begin{bmatrix} \text{Re}(s^r) & \text{Im}(s^r) \end{bmatrix}^T
$$

$$
\Gamma = \begin{bmatrix} \text{Re}(\Theta) & -\text{Im}(\Theta) \\
\text{Im}(\Theta) & \text{Re}(\Theta) \end{bmatrix}
$$

From [16], [24], robust reconstruction of a sparse signal/image under noise corrupted data can be achieved by solving the following convex optimization problem which is also referred to as Dantzig selector

$$
\hat{s} = \arg\min \| \tilde{s} \| \quad \text{s.t.} \quad \| \Gamma^T (\tilde{y} - \Gamma \tilde{s}) \|_c < \delta
$$

(20)

where $\delta$ represents a small tolerance error which can be determined using the cross validation strategy for an automatic selection of the error value in the optimization process [21].

**D. Numerical Results**

In the first example, we investigate the imaging of a rectangular PEC object behind a single layer homogeneous wall. The target is $0.3 \text{ m} \times 0.2 \text{ m}$ and is $1.5 \text{ m}$ away from the front boundary of the wall. The permittivity, conductivity and thickness of the wall are $\varepsilon_r = 6$, $\sigma = 0.05 \text{ S/m}$ and $d = 0.2 \text{ m}$. The monostatic radar system scans the region of interest at a distance $0.3 \text{ m}$ away from the wall, synthesizing a $2 \text{ m}$ aperture with $0.05 \text{ m}$ inter element spacing. The operating frequency ranges from $1 \text{ GHz}$ to $3 \text{ GHz}$ with $61$ equally spaced frequency bins. We have used background subtraction to remove the direct wall reflections [6]. Alternatively, in practice the effect of the wall reflections can be removed by either time-gating the received signal, or by analytically modeling the wall in frequency-domain and subtracting its response from the cumulative wall plus the target response [7].

Figure 3 (a) shows the imaging result using conventional back-projection beamformer in (5) using the full set of data, i.e. data measured at all frequencies and all antenna locations. For this single layer homogeneous wall, the transmission coefficient in (11) is used. If only two frequencies are randomly measured at each antenna location, imaging result using the conventional back-projection beamformer is shown in Fig. 3(a). Due to the lack of sufficient information about the target, the image is blurred, distorted, and has higher sidelobe levels, as evident from Fig. 3(b). By exploiting the sparsity of the image space and solving the sparse constraint optimization problem in (20) with the same limited amount of measurement data, the reconstructed image is given in Fig. 3 (c). From the CS through-wall imaging result we find that the targets is clearly identified and well located at the true position. From the comparison of Fig. 3 (b) and (c) it is clear that a much higher resolution and cleaner image can be obtained through sparse constraint optimization. As seen from the CS imaging result with only $3.3\%$ of the full set of data, the CS imaging approach gives even better and less cluttered image than that beamformed using full set of data in Fig. 3 (a).
In urban sensing applications, we often encounter situations to detect and identify targets inside a building with multilayered inner walls or walls separated by a hallway. This is challenging and beyond the capability of the standard back-projection beamformers using ray tracing technique, and has not been studied in the existing CS TWRI approaches reported in the literature. In the second example, we present the imaging result of multiple targets behind external and interior walls separated by a hallway, as shown in Fig. 4 (a). The width of the hallway is 1.1 m, which is sandwiched by an exterior and internal wall whose dielectric constant and thickness are \( \varepsilon_r = 7 \) and \( d = 0.15 \text{ m} \). We note that the wall parameters in such scenario can be efficiently estimated using the technique described in [41-43]. Figure 4 (b) shows the beamforming result of the two targets using the full set of data. If only two frequencies are measured at each antenna location, which results in \( 41 \times 2 \) spatial-frequency measured data, it is very difficult to identify the targets as shown in Fig. 4 (c). However, a much less cluttered and focused image can be achieved by using the generalized CS TWRI approach with the same random measured limited data, as can be seen from Fig. 4 (d).

III. THROUGH-THE-WALL IMAGING WITH MIMO RADAR

In this section, we present the detailed formulation of 2-D through-the-wall imaging with MIMO radar for real-time data acquisition. This method can be easily generalized to 3-D TWRI, if needed.

A. Problem Formulation

Figure 5 shows a 2-D scenario of through-the-wall imaging with MIMO radar. The MIMO radar system consists of \( M \) transmitting antennas located at \( [x_{tm}, z_{tm}] \) and \( N \) receiving antennas located at \( [x_{rn}, z_{rn}] \). The operating frequency ranges from \( f_{\text{min}} \) to \( f_{\text{max}} \) with a frequency step \( \Delta f \).
Under the point target model, which ignores the multiple scattering effects, the received signal can be written as

\[ E_s(\rho_m, \rho_p, \rho_r) = \int G(\rho_m, \rho_p, k_p) \frac{\sigma(\rho)}{d^2} d\rho \]  

(21)

where \( E_s(\rho_m, \rho_p, \rho_r) \) is the received scattered field at the \( m \)-th receiver location due to the illumination of the \( n \)-th transmitter, \( \sigma(\rho) \) is the reflectivity of the target, \( \rho_m, \rho_p \) and \( \rho_r \) are the position vectors of the \( m \)-th transmitter, \( n \)-th receiver and target, i.e., \( \rho_m = (x_m, z_m) \), \( \rho_p = (x_p, z_p) \), \( r = (x, z) \), \( k_p \) is the freespace wavenumber of the \( p \)-th operating frequency, \( G(\rho_m, \rho, k) \) and \( G(\rho, \rho_m, k) \) are the layered media Green’s functions, which relate the wave propagation process from the transmitter to the target and from the target to the receiver, in the presence of the wall. Then, the through-the-wall image can be reconstructed as

\[ I(\rho) = \int_{-\infty}^{\infty} dk_p \int_{-\infty}^{\infty} d\rho_m \int_{-\infty}^{\infty} d\rho_p E_s(\rho_m, \rho_p, \rho_r) \]  

(22)

In the above imaging formulation the inner two integrands correspond to the beamforming over all the transmitter and receiver locations and the outer integrand is the coherent summation over all the operating frequencies. Assume that the targets are within the far field of the antenna, the far field Green’s function of the walls can be approximated as \( G(\rho_m, \rho_p, k) \approx \frac{e^{-jk_pR_m}}{4\pi R_m} \) and \( G(\rho, \rho_m, k) \approx \frac{e^{jk_pR_m}}{4\pi R_m} \).

(23)

where \( R_m = |\rho_m - \rho| \), \( R_n = |\rho_n - \rho| \), \( T_i \) and \( T_n \) are the wall transmission coefficients from the transmitter to target and the target to receiver, respectively.

By substituting (23) into (22), the MMIO radar through-the-wall imaging formula can be derived as

\[ I(\rho) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\rho_m \int_{-\infty}^{\infty} d\rho_p E_s(\rho_m, \rho_p, k_p) \]  

(24)

In the above equation, the real-valued term \( 16\pi^2R_mR_n \) is omitted. It is found from (24) that the wall effect is efficiently taken into account and well compensated through the background media Green’s function. Due to the incorporation of the layered media Green’s function, the imaging algorithm can easily be generalized for the imaging of targets behind multilayered building walls.

### B. Numerical Results

As the first example we investigate the imaging of targets behind a single layer homogeneous wall with MIMO radar. The forward scattering data was simulated using a full wave EM solver based on finite difference time domain method (FDTD). In the simulation we used eight transmitters equally spaced from -1 m to 1 m and thirty two receivers equally spaced from -1.25 m to 1.25 m along x direction at a standoff distance of 0.3 m. The dielectric constant, conductivity and thickness of the wall are \( \varepsilon_b = 6 \), \( \sigma_b = 0.05 \) S/m, and \( d = 20 \) cm. The operating frequency ranges from 1 GHz to 3 GHz. The three target investigated in this example are rectangular, cylindrical, and square PEC objects with the designation of Target 1, Target 2, and Target 3, respectively. The spatial characteristics of the three targets are summarized in Table 1.

<table>
<thead>
<tr>
<th>Target</th>
<th>Type</th>
<th>Size</th>
<th>Center Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target 1</td>
<td>Rectangular</td>
<td>0.3 m x 0.2 m</td>
<td>(-0.55 m, -2.1 m)</td>
</tr>
<tr>
<td>Target 2</td>
<td>Cylindrical</td>
<td>R = 0.1 m</td>
<td>(0, -1.9 m)</td>
</tr>
<tr>
<td>Target 3</td>
<td>Rectangular</td>
<td>0.2 m x 0.2 m</td>
<td>(0.5 m, -1.8 m)</td>
</tr>
</tbody>
</table>

In order to demonstrate the necessity and importance of taking into account the wall effect, first a conventional freespace beamformed imaging result is shown in Figure 6 (a), where the true regions of the targets are indicated with white dashed rectangle, circle and square. From the freespace beamformed imaging result we find that the targets are not only shifted in the downrange (about 0.3 m) but also widened in the cross range. This is due to the fact that the wave propagation effect is not considered and compensated in the conventional freespace radar imaging. Figure 6 (b) shows the imaging results of the targets using the proposed MIMO through-the-wall beamformer. From this image we find that the three targets are clearly identified and well located at their true locations. It is clear from this image that, due to the proper compensation of the wall effects, high quality focused images of the targets can be achieved using the proposed beamformer for through-the-wall imaging with MIMO radar.

In the second example, we present the imaging of target behind external and interior walls separated by a hallway using MIMO radar (as shown in Figure 7 (a)), which is a representative scenario in urban sensing of building interior targets [4], [7], [24], [37]. The permittivity, conductivity and
thickness of the exterior and interior walls are $\varepsilon_b = 6$, $\sigma_b = 0.05$ S/m, and $d = 20$ cm. The width of the hallway between the exterior and interior walls is 1.2 m. The targets under investigation are square and circular cylinders with the designation of Target 1 and Target 2, respectively. The spatial characteristics of the two targets are summarized in Table 2. The radar operating conditions are the same as in the previous example.

<table>
<thead>
<tr>
<th>Table 2: Target Spatial characteristics</th>
</tr>
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<tbody>
<tr>
<td>Target Type</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>Target 1</td>
</tr>
<tr>
<td>Target 2</td>
</tr>
</tbody>
</table>

Figure 6. Imaging of targets behind a single layer wall with MIMO radar (a) freespace imaging result; (b) through-the-wall imaging result.

Figure 7 (b) shows the imaging result of the targets using freespace beamforming algorithm. The true regions of the targets are indicated by the white dashed circle and square. From the freespace imaging result we find that the targets are not correctly localized at their true locations. For multiple building walls, the targets are observed to be shifted about 0.65 m away from their true locations in the down range, which is about twice the target dimension. Figure 7 (c) shows the imaging results of the targets using the proposed MIMO through-the-wall beamformer. From the image we find that the targets are clearly identified and well located at their true locations. It is clear from this figure that the proposed MIMO beamformer is successful in the imaging of the target behind multilayered building walls without distortion or displacement, and with a significantly reduced data acquisition time as compared to conventional monostatic SAR operation.

Figure 7. Imaging of targets behind external and interior walls separated by a hallway with MIMO radar (a) simulation geometry; (b) freespace imaging result; (c) through-the-wall imaging result.

IV. GENERALIZED DT ALGORITHM FOR REAL-TIME TWRI

In [34] a 2-D diffraction tomographic algorithm for the imaging of targets behind a single layer wall, derived from the stationary phase method, has been presented. In this section, we present a generalized DT algorithm for 2-D real-time TWRI based on the exploding reflection model that is capable of imaging of targets behind either single or multilayered building walls.
A. Problem Formulation

As shown in the TWR configuration with SAR in Figure 1, the radar system moves along a scan line parallel to the wall in the observation region denoted as \( D_{\text{obs}} \). The radar operating frequency ranges from \( f_{\text{min}} \) to \( f_{\text{max}} \). The wall region consists of either single or multilayered building walls. The targets to be imaged are located in an inaccessible investigated region denoted as \( D_{\text{inv}} \) behind the walls. The investigated domain is chosen to be a rectangular region \(-X_{\text{inv}} \leq x \leq X_{\text{inv}} \) and \( z_{\text{min}} \leq z \leq z_{\text{max}} \). The relative complex permittivity \( \varepsilon_r \) inside \( D_{\text{inv}} \) can be defined as

\[
\varepsilon_r(r) = \begin{cases} \varepsilon_r, & r \in D_{\text{inv}} \\ \varepsilon_N, & \text{otherwise} \end{cases}
\]

(25)

where \( r = \hat{x} x + \hat{z} z \) is the target position vector.

Let us assume that the electric current is a 2-D point source, which is equivalent to a 3-D line source directed in the \( y \) direction. Then the scattered electric field at the receiver can be written as

\[
E_s(r_m, k) = k^2 \int_{D_{\text{inv}}} G(r_m, r, k) E_i(r, r_m, k) \chi(r) \, dr
\]

(26)

where \( r_m = \hat{x} x_m + \hat{z} z_m \) is the antenna location, \( G \) is the Green’s function for the background media, \( E_i(r, r_m, k) \) is the total electrical field inside the target, \( \chi(r) \) is the contrast function of the object

\[
\chi(r) = \varepsilon_r - 1, \quad \text{and} \quad \varepsilon_r
\]

is the relative permittivity of the object. Since the total field inside the target also depends on the contrast of the target, \( (26) \) is a nonlinear equation that is very time consuming to solve. In order to linearize the equation, the first-order Born approximation is employed, in which the total field inside the target is approximated by the incident electric field

\[
E_i(r, r_m, k) \cong E_i^{\text{inc}}(r, r_m, k) = i k \eta \varepsilon_r G(r, r_m, k)
\]

(27)

Then the received scattered field can be derived as

\[
E_s(r_m, k) = i \eta k^3 \int_{D_{\text{inv}}} G(r_m, r, k) G(r_m, r, k) \chi(r) \, dr
\]

(28)

where \( \eta \) is the wave impedance in the freespace.

Using the reciprocity property of the Green’s function

\[
G(r_m, r, k) = G(r, r_m, k)
\]

(29) can be rewritten as

\[
E_s(r_m, k) = i \eta k^3 \int_{D_{\text{inv}}} G^2(\rho, \rho, k) \chi(\rho) \, d\rho
\]

The Green’s function for the planar layered walls can be expressed in the spectral form as detailed in [21]

\[
G(\rho, \rho, k) = \frac{i}{4\pi} \int_{-\infty}^{\infty} \frac{T(k_x)}{k_{iz}} \exp\left(ik_x(x_R - x) + ik_{iz}z_R - ik_{nz}z\right) dk_x
\]

(30)

where \( T \) is the transmission coefficient for the walls, can be calculated from (8).

For derivational convenience, (29) can be written in the following operator form

\[
E_s(r_m, k) = (L \chi)(r)
\]

(31)

The operator \( L \) maps the contrast function from the object space to the data space, in particular,

\[
L(\chi) = j \eta k^3 \int_{D_{\text{inv}}} G^2(r_m, r, k) \chi(r) \, dr
\]

Then the image can be then expressed by

\[
I(r) = L^a \left( E_s(r_m, k) \right)
\]

(33)

where \( L^a \) is the adjoint operator that maps the scattered field from the data space to the object space, in particular [38], [40]

\[
L^a(\chi) = j \eta k^3 \int_{k_{iz}}^{k_{iz}} \int_{D_{\text{inv}}} G^2(r_m, r, k) \chi(r) \, dr
\]

(34)

where the * denotes the complex conjugation. The complex conjugation in the adjoint operator is essentially the back-propagation of the wave to the location where it is excited. From the concept of time reversal imaging, the complex conjugation in the frequency domain is equivalent to the time reversal in the time domain. The back-propagation of the time reversed field will focus on the target.

In this paper, to accelerate the imaging algorithm the assumption of exploding reflection model is employed [39]. In particular, the squared term of the Green’s function \( G^2(r_m, r, k) \) is approximated by \( G(r_m, r, 2k) \). Then (34) can be further simplified as

\[
L^a(\chi) = j \eta k^3 \int_{k_{iz}}^{k_{iz}} \int_{D_{\text{inv}}} G^2(r_m, r, 2k) \chi(r) \, dr
\]

(35)

Substitute the spectral form of the layered media Green’s function (30) into (11) one has

\[
L^a(E_s(r_m, k)) = \int_{\rho_R}^{\rho_L} E_s(r_m, k) e^{-ik_{nz}z_R} \, d\rho
\]

It is noticed that due to the use of the exploding model, \( k_x = \sqrt{4k_i^2 - k_z^2}, \quad i = 1, 2, \ldots, N \). Then the imaging formula can be derived as

\[
L^a(E_s(r_m, k)) = \int_{k_{iz}}^{k_{iz}} k^3 dk \int_{-\infty}^{\infty} dk_x \hat{E}_s(k_x, k) \frac{T(k_x)}{k_{iz}^3} e^{ik_x x_R - ik_{nz}z_R}
\]

(37)

where \( \hat{E}_s(k_x, k) \) is the Fourier transform of \( E_s(r_m, k) \) with respect to \( x_R \), in particular.
The above integral for \( \tilde{E}_s(k_s, k) \) can be efficiently evaluated using FFT.

B. Numerical Results

In the first example, we present the imaging of a human behind a hollow concrete wall. The measurement data was generated using XFDTD\textsuperscript{\textregistered}, a commercial full-wave electromagnetic simulator based on FDTD from Remcom Inc. The dimension of the HiFi male human model is 0.57 m \( \times \) 0.324 m \( \times \) 1.88 m and made up of 2.9 mm cubical FDTD mesh cells, 23 different tissue types with realistic dielectric and conductivity parameters. The front and side view of the human is shown in Fig. 8. The radar measures at a distance of 0.3 m from the front wall synthesizing a 2 m aperture with inter-element spacing 0.05 m. In the XFDTD simulation it took about 1.5 minutes for each transmitting location on a P4 8-core 2.8 GHz personal computer with 16G RAM memory. The operating frequency covers the range from 1-3 GHz. The inner and outer wall has the same electrical parameters with dielectric constant \( \varepsilon_r = 6 \) and conductivity \( \sigma = 0.01 \) S/m. The thicknesses of outer and inner concrete walls are 0.1 m and are the same as the air gap between the two walls.

Fig.8 Simulation geometry of a human behind a hollow concrete wall

Fig. 9 Imaging of the human behind a hollow concrete wall

The investigation domain is a 2 m \( \times \) 2 m and is divided into 100 \( \times \) 100 pixels. Figure 9 is the imaging result of the human using the proposed imaging algorithm. The approximate true region of the human is indicated with a white dashed ellipsoid. From this figure we find that through the proper incorporation of the multilayered media Green’s function, a high quality focused imaging result can be achieved using our proposed imaging algorithm and is very computationally efficient. It took only about 0.63 s in Matlab to form the image of the human on a P4 4-core 2.3 GHz CPU personal computer with 4G RAM memory. It can be 50-100 times faster than existing imaging algorithms in [6, 31, 32].

In the last example we investigate the problem of imaging of multiple targets located behind external and internal walls separated by a hallway, which is a typical scenario in urban sensing of building interior targets. The thickness of both the external and internal walls is 0.2 m with relative permittivity \( \varepsilon_r = 6 \) and conductivity \( \sigma = 0.01 \) S/m. The width of the hallway between the two walls is 1.2 m, as shown in Figure 10 (a). The radar antenna transmits an UWB EM wave covering the frequency range from 1GHz to 3GHz and measures the return signal at a distance of 0.3 m from the external wall over a synthetic aperture from -2 m to 2 m. The two targets investigated in this example are two PEC cylinders with a radius of 0.3 m and 0.2 m centred at (-0.7 m, -3.6 m) and (0.8 m, -3.3 m), respectively. The investigation domain is a 2.4 m \( \times \) 2 m rectangular region and is divided into 120 \( \times \) 100 pixels.

Fig.10 Imaging of multiple targets behind the external and interior walls separated by a hallway (a) simulation configuration; (b) imaging result

Figures 10 (b) shows the imaging results of the targets using the proposed algorithm, where the correct locations of the targets are indicated with white dashed circles. From Fig. 10 (b) we find that the two targets are clearly identified and well localized at their correct locations. It is evident from the image that, the propagation effect has been properly taken into account and high quality focused
images of the targets can be achieved using the proposed imaging algorithm. In this example, it takes only 0.67 s to form the image shown in Fig. 10 (b) when the generalized DT TWRI algorithm is used. A considerable time saving has been achieved in the process of imaging due to the utilization of FFT in the algorithm.

V. CONCLUSIONS

Conventional through-the-wall radar imaging techniques face considerable challenges, such as requirement to acquire a large amount of measured data, long data acquisition and lengthy processing time. In this paper, we present three techniques to address these challenges in TWRI: 1) compressive sensing imaging approach; 2) through-the-wall imaging with MIMO radar; 3) generalized DT algorithm for real-time TWRI. All these techniques are applicable to the imaging of targets behind either a single or multilayered building wall via the incorporation of the layered medium Green’s function that fully take into account and compensate for the wall effects. Representative results show that the proposed techniques are very efficient to reduce the long data acquisition and processing time in TWRI and can provide high quality focused imaging results in various wall-target scenarios. All these techniques can be generalized to 3-D TWRI scenarios and the results will appear in the future.

REFERENCES


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